Operational amplifier (dynamic characteristics) 1B

Exercise prepared by Bogdan Pankiewicz (English version by Miron Kłosowski)

1. Introduction

The purpose of this exercise is to familiarize yourself with the basic properties of a differential pair and a simple operational amplifier. The exercise is performed as follows:
a) measurements of the gain and input resistance of a differential pair of bipolar transistors operating in a differential and common input driving modes,
b) measurements of the harmonic distortion of a differential pair,
c) measurements of the gain, bandwidth and output resistance of a simple operational amplifier working in a non-inverting configuration.

Tested operational amplifier has a built-in frequency compensation, which reduces bandwidth.

Before starting the exercise, the student should be familiar with the course (basic information is presented in this paper). Teacher is obliged to check the preparation for the exercise.

2. Measurements

2.1 Measurements of differential pairs (circuit A)

2.1.1 Differential input mode measurements

- Use the rotary switch to select the differential pair A, then set the *DIFF/CM* button to the released position, the K_{RE} button should be also set to the released position. Measure the harmonic distortion h as a function of the input voltage V_{we} for the signal frequency equal to 1kHz.
- Select the value of the input voltage V_{we}=10mV, for which h<5% (f=1kHz) and measure the differential gain $K_r=V_{wy}/V_{we}$.
- Select the value of the input voltage V_{we}=10mV and measure the input resistance R_{INDIFF}. Input resistance is measured using additional resistor R_{SZER} connected in series with the internal resistance of the generator R_{GEN}. Press the $\boxed{R_{INDIFF}}$ button and record the output voltage. The exact description of the R_{INDIFF} measurement is located in the theoretical part of this paper.

2.1.2 Common input mode measurements

- Set the input voltage equal to 1V, frequency equal to 1 kHz, set the *DIFF/CM* button to the pressed position and measure the gain in the common mode $K_s=V_{wy}/V_{we}$.
- Set the input voltage equal to 1V, frequency equal to 1 kHz and measure the input resistance R_{INCM}. Input resistance is measured using additional resistor R_{SZER} connected in series with the internal resistance of the generator R_{GEN}. Press the $\boxed{R_{INCM}}$ button and record the output voltage. The exact description of the R_{INCM} measurement is located in the theoretical part of this paper.

2.2 Operational amplifier measurements

- Use the rotary switch to select the circuit B, then for resistor feedbacks 10k/10k, 1k/10k and 100/10k set the value of the input voltage according to the data specified in the measurement protocol, set the frequency equal to 100Hz and use the oscilloscope to measure the frequency characteristics, the 3dB cutoff frequencies and the unity-gain frequencies f_T.

3. Elaboration of the results

1) Plot a graph of nonlinear distortion *h* as the function of the input voltage V_{we} for the circuit A (differential input mode).
2) For a differential pair calculate analytically:
   - operating points of transistors,
   - input resistance in differential mode,
   - input resistance in common mode,
   - gain in differential mode,
   - gain in common mode,
   - CMRR coefficient.

   Compare the results of calculations with the results of measurements in the table in the measurement protocol.
3) Plot the measured characteristics of the amplifier (20 log $|K_u|$) for feedback resistors 10k/10k, 1k/10k and 100/10k on separate graphs (linear vertical axis, logarithmic horizontal axis). Mark the 3dB cutoff frequencies and the unity-gain frequency f_T.

Write your own conclusions and observations. Compare circuits and write comments on differences between calculations and measurements.

4. Theory

All circuits measured in the lab have built-in input and output buffers with parameters shown in the following table:

Parameter	Unit	Value
Gain	V/V	1 , -1
Input resistance R_{BUF}	MΩ	1
Output resistance R_{GEN}	Ω	50
Input capacitance C_{BUF}	pF	3
Cutoff frequency	MHz	4

The Π-type and T-type bipolar transistor models shown on Fig. 1 and 2 respectively are used in the theoretical part of the paper.

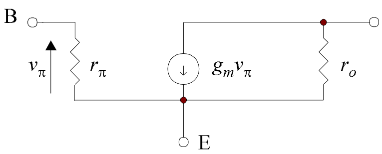

Fig. 1 Π-type bipolar transistor model.

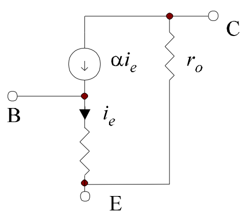

Fig. 2 T-type bipolar transistor model.

The small-signal model parameters:

$$gm = \frac{I_C}{V_T},\ r_\pi = \frac{\beta}{gm},\ \alpha = \frac{\beta}{\beta+1},\ r_e = \frac{V_T}{I_E} \tag{1}$$

4.1 Bipolar differential pair (circuit A)

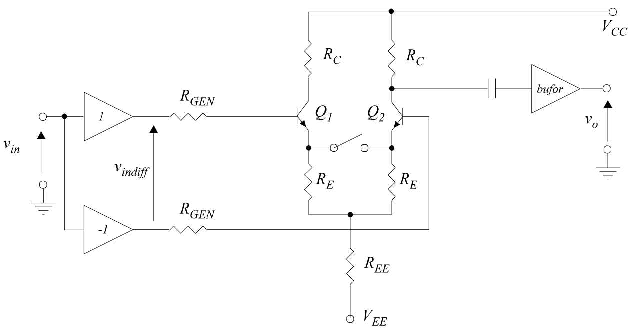

Fig. 3 Schematic diagram of the differential pair measured in the exercise.

4.1.1 Operating point

The input buffers provide a constant voltage at the output equal to 0V. Because the R_{GEN} resistances are low it can be assumed that the DC voltages on the gates are also equal to zero. Thus, the current flowing through the resistor R_{EE} is equal to:

$$I_{EE} = \frac{-V_{EE} - V_{BE}}{R_{EE} + \frac{R_E}{2}} \tag{2}$$

while the emitter currents of transistors Q_1 and Q_2 are equal to half of the resistor's R_{EE} current. Voltages on the individual electrodes of elements Q_1 and Q_2 can be determined according to the formula:

$$V_B = 0\,,\ V_E = -V_{BE} \approx 0.7V\,,\ V_C = V_{CC} - R_C I_C\,, \tag{3}$$

where $I_C = \alpha I_E$, $\alpha = \frac{\beta}{\beta + 1}$. (4)

The small-signal model parameters of transistors can be calculated using the following formulas:

$$gm = I_C / V_T\,,\ r_\pi = \beta / gm \tag{5}$$

for Π-type model and

$$r_e = V_T / I_E \tag{6}$$

for T-type model, where $V_T = kT/q$ is a thermal voltage (approximately 25.8mV at room temperature), k – Boltzmann's constant, T – absolute temperature, q - elementary charge.

4.1.2 Small-signal operation of the differential pair

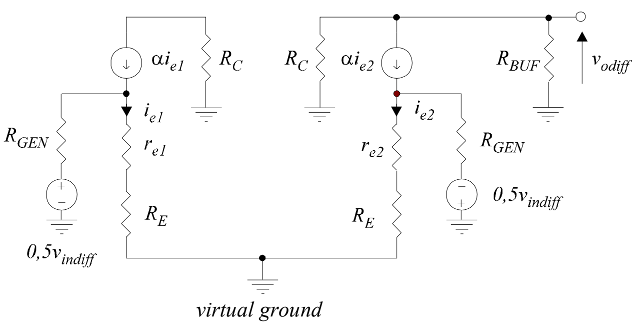

Fig. 4 Small-signal equivalent circuit of the differential pair from Fig. 3 for differential mode input. Output resistance of the transistors have been neglected.

The differential gain and input resistance can be determined by using the schematic diagram of the Fig. 4 to obtain the equations:

$$\frac{v_{odiff}}{v_{in}} = 2\frac{v_{odiff}}{v_{indiff}} = \frac{R_{INDIFF}}{R_{INDIFF} + 2R_{GEN}}\alpha\frac{R_C \| R_{BUF}}{R_E + r_e} \tag{7}$$

$$R_{INDIFF} = 2(\beta + 1)(R_E + r_e) \text{ where } r_{e1} = r_{e2} = r_e \tag{8}$$

because of the equality of currents biasing pair of differential transistors.

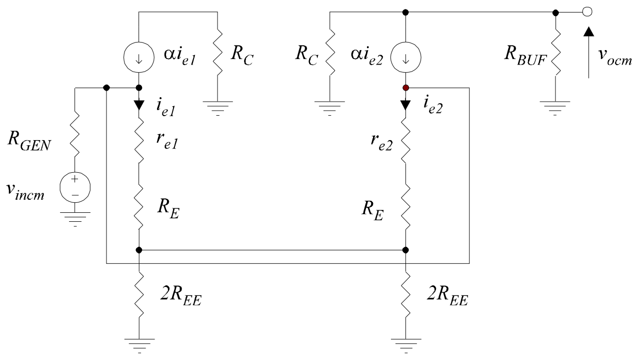

Fig. 5 Small-signal equivalent circuit of the differential pair from Fig.3 for common mode input. (I.e. using only the upper buffer and short-circuit between the two bases of bipolar transistors). Output resistance of the transistors have been neglected.

Based on Fig. 5 gain and input resistance in common mode are respectively equal to:

$$\frac{v_{ocm}}{v_{incm}} = -\frac{R_{INCM}}{R_{INCM} + R_{GEN}}\alpha\frac{R_C \| R_{BUF}}{2R_{EE} + R_E + r_e} \tag{9}$$

$$R_{INCM} = \frac{1}{2}(\beta + 1)(2R_{EE} + R_E + r_e) \text{ where } r_{e1} = r_{e2} = r_e \tag{10}$$

For differential amplifiers a parameter called the Common Mode Rejection Ratio is defined, which is indicated by an acronym CMRR and calculated by the formula (11).

$$CMRR = 20\log\left(\frac{\frac{v_{odiff}}{v_{indiff}}}{\frac{v_{ocm}}{v_{incm}}}\right) \tag{11}$$

Short-circuit between emitters of transistors Q_1 and Q_2 (K_{RE} switch in the pressed position) does not change the operating point but affects the small-signal properties of the differential pair for differential input mode (i.e. for the calculation assume R_E=100Ω in formulas: (2), (9) and (10) while R_E=0Ω in formulas: (7) and (8)).

4.2 Operational amplifier (circuit B)

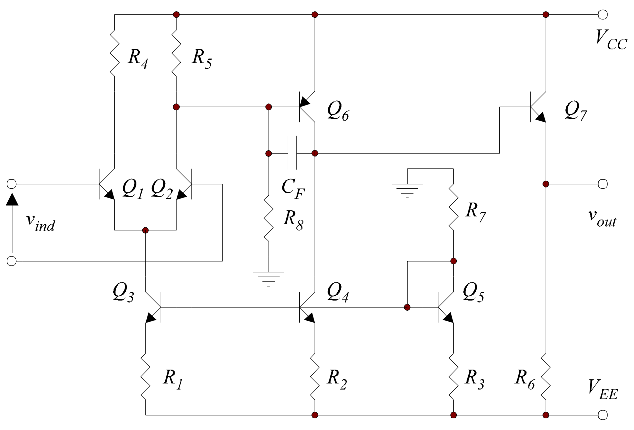

Fig. 6 Schematic diagram of the operational amplifier measured in the exercise.

The transistors Q_1-Q_5 of the amplifier of Fig. 6 are identical and come from the UL1111 integrated circuit. Elements Q_1 and Q_2 form a differential input pair, Q_6 is an amplifier stage in CE configuration, while the transistor Q_7 is an output buffer stage in the CC configuration. Other transistors form a bias circuits. The capacitor C_F is introduced for the frequency compensation.

4.2.1 Operating point

The operating point of the amplifier of Fig. 6 is calculated with the assumption that it is working in a negative feedback loop (it is not shown in Fig. 6, whereas it is shown in Fig. 9). Because of the high loop gain, constant output voltage of the amplifier at zero input voltage will be approximately zero. Neglecting the base currents of transistors Q_3-Q_5 we get the following formula:

$$I_{C3} = I_{C5} = \frac{-V_{CC} - V_{BE}}{R_7 + R_3} \quad (12)$$

The current of the transistor Q_4 (due to the unequal resistance of the elements R_2 and R_3) must be calculated iteratively [1, p. 41] using the following equation:

$$I_{C4}^{(n+1)} = \frac{V_T}{R_2} \ln\left(\frac{I_{C5}}{I_{C4}^{(n)}}\right) + \frac{R_3}{R_2} I_{C5} \quad (13)$$

where $I_{C4}^{(n+1)}$ is calculated using the previous value $I_{C4}^{(n)}$ until the difference between $I_{C4}^{(n+1)}$ and $I_{C4}^{(n)}$ becomes negligible. The collector current of the transistor Q_6 is equal to the sum of the collector current of Q_4 and the base current of Q_7. Assuming that a DC component of the output voltage is equal to zero (due to a feedback) we get the following formula:

$$I_{E7} = -V_{EE} / R_6 \quad (14)$$

hence,

$$I_{C6} = I_{C4} + I_{E7} / (\beta_7 + 1) \quad (15)$$

The emitter currents of differential pair transistors are equal to half of the Q_3 collector current.

4.2.2 Simplified small-signal analysis

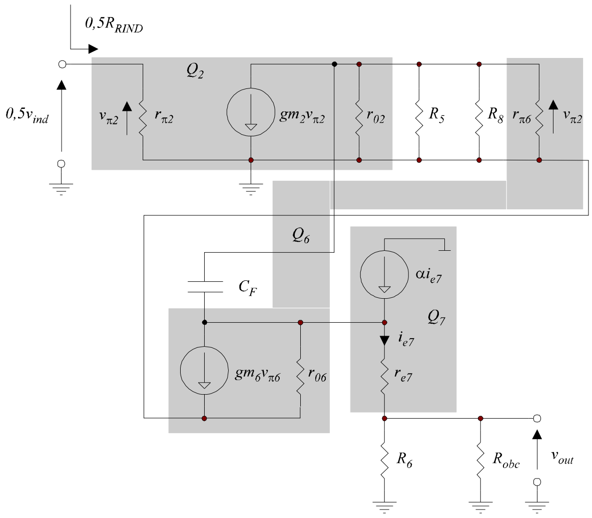

Fig. 7 Small-signal equivalent circuit of the amplifier from Fig. 6 for differential mode input. It is assumed that the amplifier is loaded with resistance R_{obc}.

Fig. 7 shows a simplified equivalent circuit of the amplifier of Fig. 6. The resistances of current sources formed by transistors Q_3-Q_5 have been neglected. However, due to the resistors in the emitters of the circuits of Q_3-Q_5 elements, a small error is introduced. Another simplification is to neglect all the parasitic capacitances of the transistors and the assumption that the bandwidth of the amplifier is limited primarily by the compensating capacitor C_F (in the case of operational amplifier it is often used to assure the stability of the closed-loop [2], [3]). The resultant amplifier consists of three stages: two with CE configurations and output with the CC configuration. On the basis of Fig. 7 mid-band gain, input and output resistances can be calculated using the following formulas, respectively:

$$A = \frac{v_{out}}{v_{ind}} = \frac{1}{2} gm_2 r_{02} \| R_5 \| R_8 \| r_{\pi 6} * \\ gm_6 \left[r_{06} \| \left[(\beta_7 + 1)(r_{e7} + R_6 \| R_{OBC}) \right] \right] \frac{R_6 \| R_{OBC}}{R_6 \| R_{OBC} + r_{e7}} \quad (16)$$

$$R_{IND} = 2 r_{\pi 2} \quad (17)$$

$$R_{OUT} = R_6 \| \left(r_{e7} + \frac{r_{06}}{\beta_7 + 1} \right) \quad (18)$$

Due to Miller's theorem the capacitance C_F can be converted to an equivalent capacitances: C_{M1} and C_{M2} and connected to earth and respectively to the base and collector nodes of Q_6 transistor [1]. The values of these capacities can be calculated using the following formulas:

$$C_{M1} = C_F (1 - K) \quad (19)$$

$$C_{M2} = C_F (1 - 1/K) \quad (20)$$

where $K = -gm_6 r_{06} \| \left((\beta_7 + 1)(r_{e7} + R_6 \| R_{OBC}) \right)$ is the gain of the Q_6 transistor stage. The poles of the transmittance created by the existence of capacitances: C_{M1} and C_{M2} can be determined from the following equations:

$$\omega_{p1} = \frac{1}{C_{M1} (r_{\pi 6} \| r_{02} \| R_5 \| R_8)} \quad (21)$$

$$\omega_{p2} = \frac{1}{C_{M2} \left[r_{06} \| \left((\beta_7 + 1)(r_{e7} + R_6 \| R_{OBC}) \right) \right]} \quad (22)$$

Given the fact that the value of the gain K reaches high values and analyzing the relations (19) - (22) ω_{p1} turns out to be the dominant pole (ω_{p1} is 2-3 orders of magnitude lower than ω_{p2}). Thus, the amplifier of Fig. 6 can be substituted with good accuracy by the equivalent voltage controlled voltage source with transmittance:

$$T(s) = \frac{V_{OUT}(s)}{V_{IND}(s)} \approx A \frac{\omega_{p1}}{s + \omega_{p1}} \quad (23)$$

and the input and output resistances defined by (17) and (18) respectively.

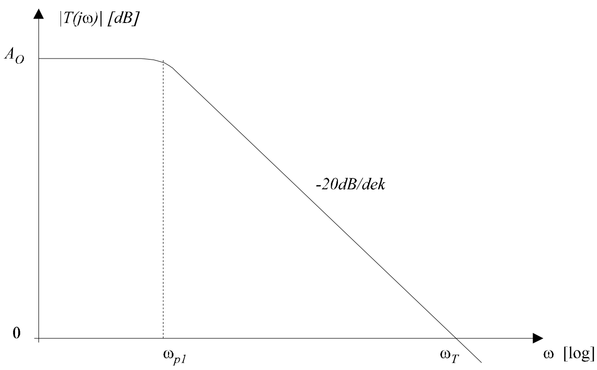

Fig. 8 Gain magnitude of the operational amplifier plotted using formula (23).

The magnitude of the open-loop amplifier transmittance, plotted on the basis of the formula (23) is shown in Fig. 8. For operational amplifiers a parameter called unity-gain angular frequency ω_T is defined. It is a frequency for which the magnitude of the voltage gain decreases to a value of 1 V/V (or 0 dB in a logarithmic scale). Converting formula (23) we get:

$$\omega_T = \omega_{p1} \sqrt{A^2 - 1} \Big|_{A >> 1} \approx A \omega_{p1} \quad (24)$$

4.2.3 Closed-loop amplifier operation

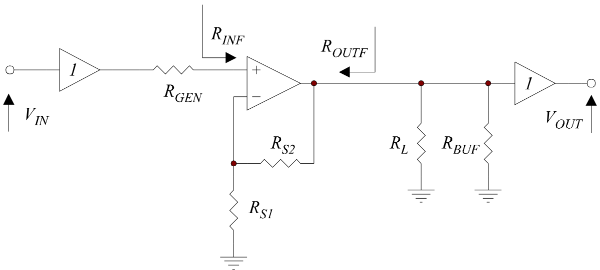

Fig. 9 The amplifier of Fig. 6 in the non-inverting configuration.

In this exercise the amplifier in the non-inverting configuration shown in Fig. 9 is measured. For the calculations we use a method based on the decomposition of the amplifier to two two-port networks: A - the operational amplifier and B – the feedback. The feedback block comprises two resistors: R_{S1} and R_{S2} which may be selected from the three available sets. This is the voltage-series feedback. Fig. 10 shows the separated two-port networks: modified amplifier A' in Fig. 10 (a) and the feedback B in Fig. 10 (b).

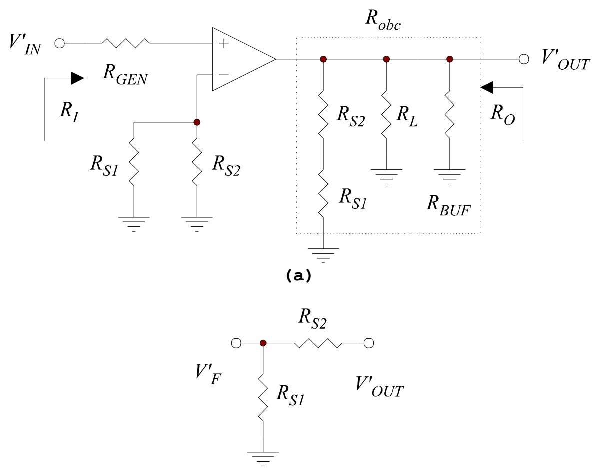

Fig. 10 Modified amplifier A' (a) and feedback two-port network B (b) of Fig. 9 circuit.

Transmittances of the modified circuit A' and the two-port network B can be calculated from the following formulas:

$$A'(s) = \frac{V'_{OUT}(s)}{V'_{IN}(s)} = A\frac{\omega_{p1}}{s+\omega_{p1}}\frac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}} \tag{25}$$

$$B = \frac{v'_f}{v'_o}\frac{R_{S1}}{R_{S1}+R_{S2}} \tag{26}$$

where A is expressed by (16) assuming that $R_{OBC} = R_{BUF}||R_L||\left(R_{S1}+R_{S2}\right)$

while the input and output resistances of the A' can be determined from the following equations, respectively:

$$R_I = R_{IND} + R_{GEN} + \left(R_{S1}||R_{S2}\right) \tag{27}$$

$$R_O = R_{OUT}||R_L||\left(R_{S1}+R_{S2}\right)||R_{BUF} \tag{28}$$

The transmittance of the closed-loop circuit can be expressed by the following formula:

$$A_F(s) = \frac{A'(s)}{1+A'(s)B} = \tag{29}$$
$$= \frac{\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}A\omega_{p1}}{s+\omega_{p1}\left(1+\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}AB\right)}$$

which gives a low-pass function with the gain for low frequency:

$$A_O = \frac{\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}A}{1+\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}AB} \tag{30}$$

and the 3dB cutoff frequency:

$$\omega_{3dB} = \omega_{p1}\left(1+\frac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}AB\right) \tag{31}$$

The input and output resistances of the closed-loop circuit in the pass-band can be calculated from the following formulas, respectively:

$$R_{INF} = R_{IF} - R_{GEN} \tag{32}$$

$$R_{OUTF} = \frac{R_{OF}R_L}{R_L - R_{OF}} \tag{33}$$

where $R_{IF} = R_I\left(1+\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}AB\right)$ and

$$R_{OF} = \frac{R_O}{1+\dfrac{R_{IND}}{R_{IND}+R_{GEN}+R_{S1}||R_{S2}}AB}$$

In the case the conditions: $R_{IND} >> R_{GEN} + R_{S1}||R_{S2}$ and $R_{OUT} << R_{obc}$ are met (for example for μA741 amplifier: $R_{IND} \approx 2\,M\Omega$ and $R_{OUT} \approx 200\Omega$) the expressions for the gain and bandwidth of the amplifier in the non-inverting configuration can be simplified to the following form:

$$A_o = \frac{A}{1+AB} = \frac{A}{1+A\dfrac{R_{S1}}{R_{S1}+R_{S2}}}\Bigg|_{A>>\frac{R_{S1}+R_{S2}}{R_{S1}}} = 1+\frac{R_{S2}}{R_{S1}} \tag{34}$$

$$\omega_{3dB} = \omega_{p1}\left(1+AB\right) = \omega_{p1}\left(1+A\frac{R_{S1}}{R_{S1}+R_{S2}}\right) \quad \text{where} \tag{35}$$

$$A = \frac{1}{2}gm_2 r_{02}||R_5||R_7||r_{\pi6}* \quad \text{and} \tag{36}$$
$$gm_6 r_{06}||\left(\left(\beta_7+1\right)\left(r_{e7}+R_6||R_L\right)\right)\frac{R_6}{R_6+r_{e7}}$$

$$\omega_{p1} = \frac{1}{C_F\left(1+gm_6 r_{06}||\left(\left(\beta+1\right)\left(r_{e7}+R_6\right)\right)\right)\left(r_{\pi6}||r_{01}||R_5||R_7\right)} \tag{37}$$

4.3 Measurement of the input resistance of the amplifiers

The input resistance is measured using an additional resistor R_{SZER} connected in series with the internal resistance of the generator. During normal operation it is bridged by a switch located on the front panel. After pressing the button marked $\boxed{R_{INDIFF}}$ (or $\boxed{R_{INCM}}$) the resistor R_{SZER} is no longer bridged (which leads to a reduction of gain).

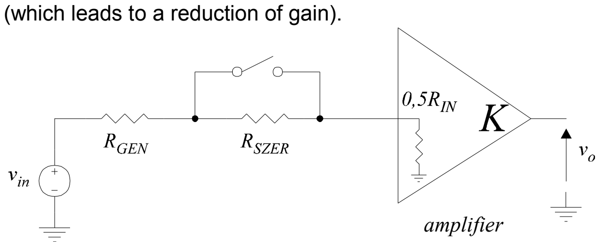

Fig. 11 Measurement method of the input resistance of the amplifier. The image shows the half of the input stage of the differential amplifier.

Marking the output voltages with bridged and present R_{SZER} resistor as v_0 and $v_0^{'}$ respectively, we get:

$$v_o = K\cdot\frac{0{,}5R_{INDIFF}}{0{,}5R_{INDIFF}+R_{GEN}}\cdot v_{in} \tag{38}$$

$$v_o^{'} = K\cdot\frac{0{,}5R_{INDIFF}}{0{,}5R_{INDIFF}+R_{GEN}+R_{SZER}}\cdot v_{in} \tag{39}$$

$$\frac{v_o}{v_o^{'}} = \frac{0{,}5R_{INDIFF}+R_{GEN}+R_{SZER}}{0{,}5R_{INDIFF}+R_{GEN}} \tag{40}$$

$$\boxed{R_{INDIFF} = 2\frac{v_o^{'}}{v_o - v_o^{'}}\cdot R_{SZER} - R_{GEN}} \tag{41}$$

In the case of the measurement of the common mode input resistance formula (41) is converted to the following form:

$$\boxed{R_{INCM} = \frac{v_o^{'}}{v_o - v_o^{'}}\cdot R_{SZER} - R_{GEN}} \tag{41b}$$

4.4 Measurement of the output resistance of the amplifiers

The output resistance is measured using additional resistor $R_{RÓW}$ connected in parallel with the load resistance of the amplifier R_L . During normal operation the resistor $R_{RÓW}$ is disconnected. During the resistance measurement this resistor is connected by the switch located on the front panel and marked $\boxed{R_{OUTF}}$.

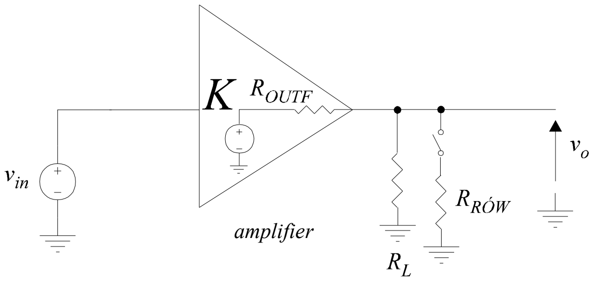

Fig. 12 Measurement method of the output resistance of the amplifier.

Marking the output voltage with disconnected and connected $R_{RÓW}$ resistor as v_0 and v_0' respectively, we get:

$$v_o = K \cdot \frac{R_L}{R_L + R_{OUTF}} \cdot v_{in} \qquad (42)$$

$$v_o' = K \cdot \frac{R_L \| R_{RÓW}}{R_L \| R_{RÓW} + R_{OUTF}} \cdot v_{in} \qquad (43)$$

$$\frac{v_o}{v_o'} = \frac{R_L \| R_{RÓW} + R_{OUTF}}{R_L \| R_{RÓW}} \frac{R_L}{R_L + R_{OUTF}} \qquad (44)$$

$$\boxed{R_{OUTF} = \frac{R_L\left(1 - \frac{v_0}{v_0'}\right)}{\frac{v_0}{v_0'} - \frac{R_L}{R_L \| R_{RÓW}}}} \qquad (45)$$

4.5 Parameters of the elements and transistors in different circuit configurations

Circuit A (differential pair):

Element/parameter	Unit	Value
Q1-Q2 β	A/A	120
R_{EE}	Ω	10k
R_E	Ω	100
R_C	Ω	5.1k
R_{SZER} differential mode	Ω	5.1k
R_{SZER} common mode	Ω	1.5M
V_{CC}	V	12
V_{EE}	V	-12

Circuit B (operational amplifier):

Element/parameter	Unit	Value
Q_1-Q_5 β	A/A	120
Q_1-Q_5 V_A	V	150
Q_6 β	A/A	370
Q_6 V_A	V	150
Q_7 β	A/A	220
Q_7 V_A	V	150
R_1	Ω	33k
R_2	Ω	1k
R_3	Ω	33k
R_4	Ω	51k
R_5	Ω	51k
R_6	Ω	1k
R_7	Ω	510k
R_8	Ω	2.2M
R_L	Ω	1k
$R_{RÓW}$	Ω	100
R_{S1}	Ω	100/1k/10k
R_{S2}	Ω	10k
C_F	pF	100
V_{CC}	V	12
V_{EE}	V	-12

Bibliography:

[1] Z. J. Staszak, J. Glinianowicz, D. Czarnecki, „Materiały pomocnicze do przedmiotu Układy elektroniczne liniowe".

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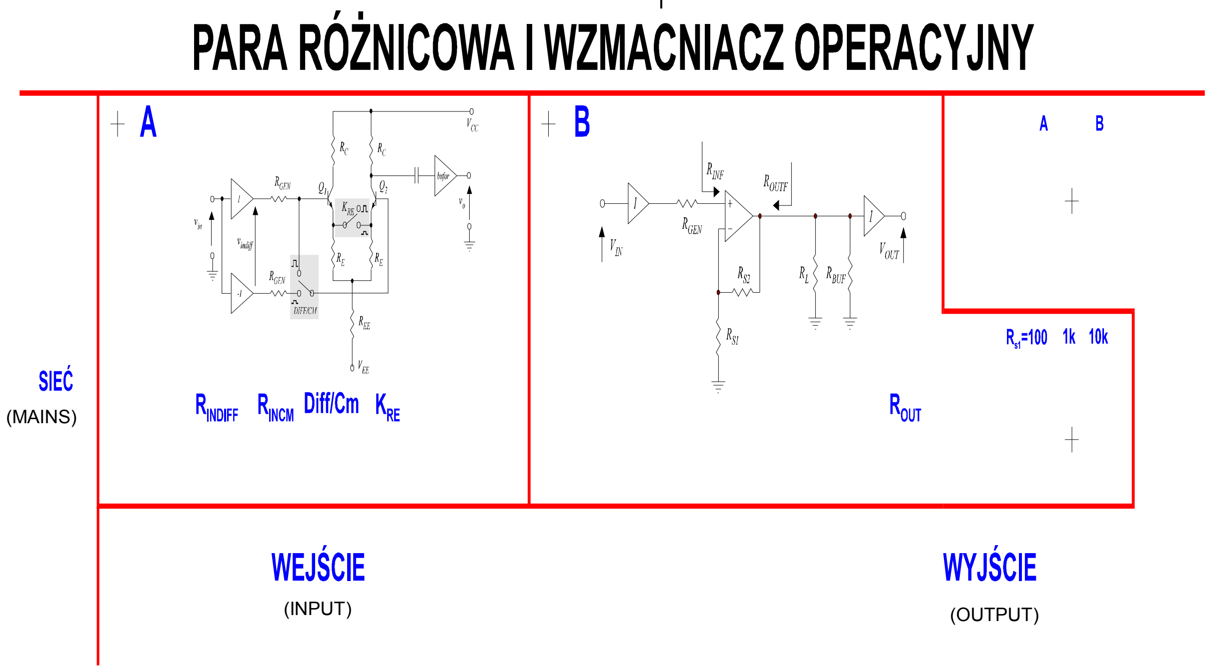

Fig. 13 Picture of the front panel.