Negative feedback 2

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1. Introduction

This exercise allows you to measure and compare the properties of an amplifier operating in the following configurations (see Fig. 21):
A – amplifier with open feedback loop (i.e. without feedback),
B – amplifier with closed feedback loop of type (1),
C – amplifier with closed feedback loop of type (2).

The tested circuit is a two-stage amplifier composed of two bipolar transistors operating in common emitter (CE) configuration. This amplifier is covered by a negative voltage-series feedback.

In this exercise, you should measure the following amplifier's parameters: gain in the centre of the passband, input and output resistance, low and high 3-decibel (3-dB) frequency. Particular configurations (A, B or C) are selected by means of rotary switch (Fig. 21). For independence of parameters of measuring instruments and quality of connections, the amplifier has built-in unity-gain buffers at both the input and the output.

Before exercise, **you should be familiar with the course (basic information is presented in this paper). Teacher is obliged to examine the preparation for the exercise.**

2. Measurements

For each of the circuits A, B, and C, **you should**:

a) Measure the **low and high 3-dB cutoff frequencies, f_{L3dB} and f_{H3dB}**. The measurement should be performed as follows:
- Set the input frequency of 18 kHz and set the input voltage (v_S) for the circuit A ≈ 1 mV$_{RMS}$, for B ≈ 10 mV$_{RMS}$, and for C ≈ 5 mV$_{RMS}$. Next, you must adjust the input voltage v_S so that the output voltage v_O is equal to 300 mV$_{RMS}$.
- Increase the generator frequency until the output voltage v_O drops to 300 mV$_{RMS}$/√2 ≈ 212 mV$_{RMS}$ — the resulting frequency is the high 3-dB cutoff frequency (Warning: During the frequency change, you must keep a constant input rms voltage!),
- Decrease the generator frequency until the output voltage v_O drops to 300 mV$_{RMS}$/√2 ≈ 212 mV$_{RMS}$ — the resulting frequency is the low 3-dB cutoff frequency (Warning: During the frequency change, you must keep a constant input rms voltage!).

b) Calculate a **center frequency f_0** from the formula $f_0 = \sqrt{f_{L3dB} \cdot f_{H3dB}}$ and measure the **amplifier gain** ($K_u = v_O/v_S$) at this center frequency.

c) Measure an **input resistance R_{in}**. Set the input frequency of 18kHz and the amplitude as in the point a). Input resistance is measured using additional resistor R_S' connected in series with the internal resistance R_S of the generator. Press the button R_{in} and note the output voltage. For more details, see the description in section 5.

d) Measure an **output resistance R_{out}**. Set the input frequency of 18kHz and the amplitude as in the point a). Output resistance is measured using additional resistor R_L' connected in parallel with the load resistance R_L of the amplifier. Press the button R_{out} and note the output voltage. For more details, see the description in section 6.

e) Measure **nonlinear distortions *h***. Set the input frequency of 18kHz and set the amplitude as in the point a).

f) Measure **the amplitude characteristic** by a point-by-point method.

3. Results elaboration

Plot the measured amplitude characteristics on separate graphs. The vertical axis should be "Gain" expressed in decibels, i.e., 20log|K_u|. The horizontal axis (i.e., frequency) should be logarithmic.

Calculate (on the basis of formulas included in the protocol) for circuits A, B, and C, the following quantities:
- operating points of transistors,
- amplifier gain K_u,
- input and output resistances of the amplifier.

Compare the above theoretical results with the measured results. Comment compliance of theoretical calculations with the measurements.
Include your conclusions.

4. Theory

4.1 Properties of negative feedback

When applied to an amplifier, a negative feedback causes:
- reduction of gain sensitivity to temperature, supply, etc.,
- modification of the input and output resistance,
- reduction of nonlinear distortion (in most cases),
- widening of the pass band (increasing the bandwidth).

Disadvantages of negative feedback are:
- it reduces an amplifier gain,
- it may cause instability of an amplifier.

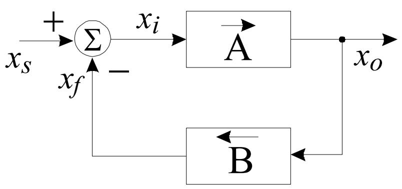

Fig. 1. Block diagram of an ideal feedback loop.

The diagram of an ideal feedback is shown in Fig. 1. The gain of the circuit with closed feedback loop is:

$$A_f \equiv \frac{x_o}{x_s} = \frac{x_o}{x_i + x_f} = \frac{Ax_i}{x_i + ABx_i} = \frac{A}{1+AB}$$

AB - loop gain
1+AB=F - amount of feedback

The impact of negative feedback on the frequency response of a linear amplifier A will be analyzed as follows. Let's assume that the frequency response of an amplifier without feedback is:

$$A(s) = \frac{A_M}{(1+\omega_L/s)(1+s/\omega_H)}$$

The amplifier gain for medium pulsations ($\omega_L \ll \omega \ll \omega_H$) is A_M, and its bandwidth is limited by two border pulsations, ω_L – low pulsation and ω_H – high pulsation. Furthermore, we assume that the characteristic of the feedback network B is constant and independent of frequency. After closing the feedback loop we get:

- the gain within bandwidth (i.e., the gain at center frequency): $A_f = \dfrac{A_M}{1 + A_M B} = \dfrac{A_M}{F}$,

- low 3-dB pulsation: $\omega_{Lf} = \dfrac{\omega_L}{(1 + A_M B)} = \dfrac{\omega_L}{F}$,

- high 3-dB pulsation: $\omega_{Hf} = \omega_H (1 + A_M B) = \omega_H F$.

As follows from the above relationships, the negative feedback expands the amplifier bandwidth. For the amplifier described by one-pole transfer function, its unity-gain frequency (the product of the gain and bandwidth of the amplifier) remains unchanged.

A valuable feature of the feedback is to reduce the gain sensitivity to changes in supply, temperature etc.

The gain sensitivity to variation in supply voltage is given by the formula:

$$S_{Uz}^{A} = \frac{\dfrac{dA}{A}}{\dfrac{dUz}{Uz}} = \frac{dA}{dUz}\frac{Uz}{A}$$

or for finite increments by the formula: $S_{Uz}^{A} = \dfrac{\Delta A}{\Delta Uz}\dfrac{Uz}{A}$

An important parameter of the whole system from Fig. 1 is its gain sensitivity to variation in the gain of a subsystem A. This sensitivity is expressed by the formula:

$$S_{A}^{A_f} = \frac{\Delta A_f}{\Delta A}\frac{A}{A_f} = \frac{1}{1 + AB} = \frac{1}{F}$$

or by the formula (useful for measurements):

$$S_{A}^{A_f} = \frac{S_{Uz}^{A_f}}{S_{Uz}^{A}}$$

because $\dfrac{\Delta A_f}{A_f} = S_{Uz}^{A_f}\dfrac{\Delta Uz}{Uz} \qquad \dfrac{\Delta A}{A} = S_{Uz}^{A}\dfrac{\Delta Uz}{Uz}$

In this exercise, we use an amplifier with a voltage-series negative feedback. Hence, only this kind of feedback will be discussed in the following text. The topology of this feedback is depicted in Fig. 2. The output voltage v_o is fed (through a two-port feedback network) to the input, in series with the source v_s. In the theoretical analysis, we assume:

a) the circuits A and B are unilateral,
b) the circuit A is a voltage amplifier with a gain v_o/v_i = A, with an input resistance R_i and output resistance R_o,
c) the circuit B has a voltage transfer function B = v_f/v_o, and it doesn't load the circuit A (i.e., doesn't change the gain of circuit A).

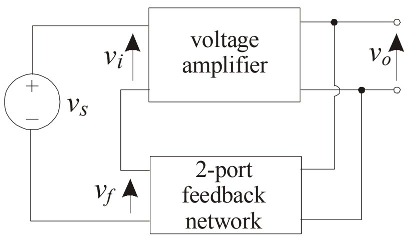

Fig. 2. Topology of voltage-series negative feedback.

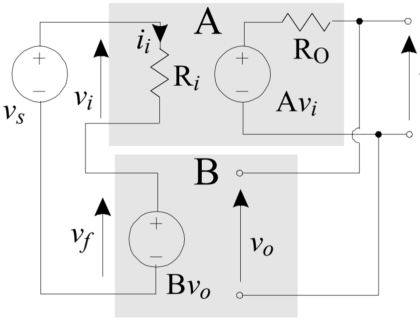

Fig. 3. Model of an amplifier with voltage-series negative feedback.

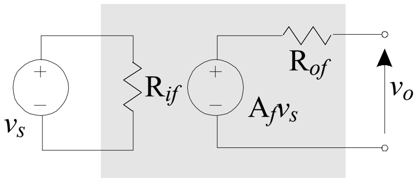

Fig. 4. Model of the main amplifier.

The circuit gain is:

$$A_f = \frac{v_o}{v_s} = \frac{A}{1 + AB}$$

The input resistance is:

$$R_{if} = R_i (1 + AB)$$

The output resistance is:

$$R_{of} = \frac{R_o}{(1 + AB)}$$

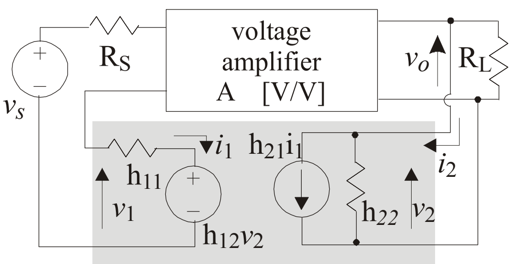

Fig. 5. Model of a real amplifier with voltage-series negative feedback.

In real implementations, two-port feedback network is mostly a passive circuit. Therefore, we can assume:

- the reverse transfer function of a two-port feedback network is negligibly small compared to the gain of an active circuit A (i.e., h_{21}=0),
- the forward transfer function of two-port feedback network is B=h_{12}=v_1/v_o at i_1=0.

The ideal two-port network A should be complemented with additional elements, that is, the resistances contributed by the feedback network, i.e., elements h_{11} and $1/h_{22}$. The resistance R_s of an input source and the load resistance R_L, should also be taken into account (R_s and R_L should be contributed to A). As a result, the diagram from Fig. 5 is converted to the diagram in Fig. 6, where the two-port network A is replaced by a modified two-port network A'. We can see, that the elements h_{11}, $1/h_{22}$, R_s and R_L are included in the network A'.

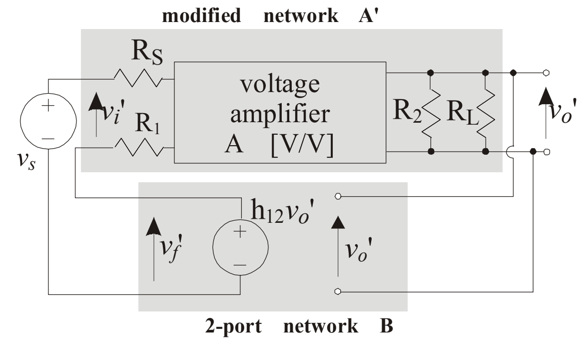

Fig. 6. The modified circuit.

As a summary, the entire process of the network transformation is depicted in Fig. 6.

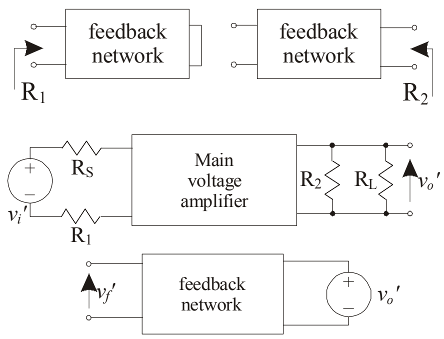

Fig. 6. Summary of transformations.

$$A' = \frac{v'_o}{v'_i} \qquad B = \frac{v'_f}{v'_o} \qquad A_f = \frac{v_o}{v_s} = \frac{A'}{1 + A'B}$$

4.2. Small-signal analysis of a two-stage amplifier with voltage-series negative feedback.

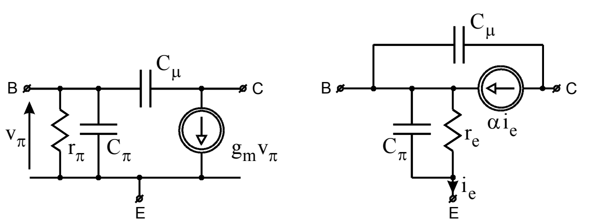

Fig. 7. Small-signal models of bipolar transistor. Left picture – model π type. Right picture – model T type.

Parameters of the transistor models from Fig. 7 are:

$$g_m = \frac{I_C}{V_T}, \qquad r_\pi = \frac{\beta}{g_m}, \qquad \alpha = \frac{\beta}{\beta + 1}, \qquad r_e = \frac{r_\pi}{\beta + 1},$$

$$C_\pi = \frac{g_m}{2\pi \cdot f_T} - C_\mu$$

4.2.1 Operating point

Operating points of the transistors can be derived under the following assumptions: DC base current I_B is very small and can be omitted (i.e., $I_B = 0$), base-emitter voltage is constant and equal to 0.7 V, thermal voltage is V_T = 25 mV. The operating point is the same for the amplifier without and with the feedback.

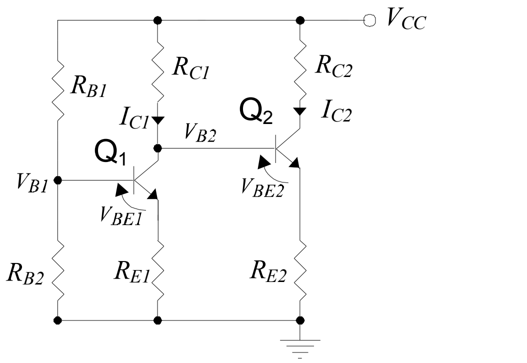

Fig. 9. Amplifier diagram for calculating of an operating point.

DC currents and voltages in the amplifier network (Fig. 9) can be calculated using the following equations:

$$V_{B1} \approx V_{CC} \frac{R_{B2}}{R_{B1} + R_{B2}} \tag{1}$$

$$I_{C1} \approx I_{E1} = \frac{V_{B1} - 0{,}7V}{R_{E1}} \tag{2}$$

$$V_{CE1} = V_{CC} - \left(R_{C1} + R_{E1}\right) I_{C1} \tag{3}$$

$$V_{B2} = V_{CC} - R_{C1} I_{C1} \tag{4}$$

$$I_{C2} \approx I_{E2} = \frac{V_{B2} - 0{,}7V}{R_{E2}} \tag{5}$$

$$V_{CE2} = V_{CC} - \left(R_{C2} + R_{E2}\right) I_{C2} \tag{6}$$

4.3. Small-signal analysis of two-stage amplifier with an open feedback loop (configuration A).

<u>Analysis for middle frequencies (i.e., near centre of bandwidth)</u>
A small-signal diagram of the amplifier from Fig. 9 is depicted in Fig. 10. The diagram is created assuming the amplifier operates in middle frequency range. In this case, all coupling capacitors are assumed to be short-circuits for AC signals, all parasitic capacitances of transistors are open, and a supply line is assumed to be grounded.

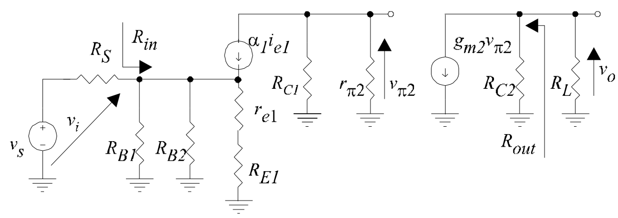

Fig. 10. Small-signal diagram of an amplifier for middle frequencies.

The input and output resistances of the considered amplifier are as follows:

$$R_{in} = R_{B1} \| R_{B2} \| (r_{e1} + R_{E1})(\beta_1 + 1) \tag{7}$$

$$R_{out} = R_{C2} \tag{8}$$

and the amplifier gain is:

$$\frac{v_o}{v_s} = \frac{v_o}{v_{\pi 2}} \frac{v_{\pi 2}}{i_{e1}} \frac{i_{e1}}{v_i} \frac{v_i}{v_s} \tag{9}$$

$$\boxed{\frac{v_o}{v_s} = g_{m2}\left(R_{C2} \| R_L\right) \frac{\alpha (R_{C1} \| r_{\pi 2})}{r_{e1} + R_{E1}} \frac{R_{in}}{R_{in} + R_S}} \tag{10}$$

<u>Analysis for high frequencies (i.e., near a high 3-dB frequency)</u>

A small-signal diagram of the amplifier of Fig. 9 is depicted in Fig. 11. The diagram is created assuming that the amplifier operates in high frequency range, i.e., near high 3-dB frequency. The high 3-dB frequency can be derived using a time-constant method. The time constants result from parasitic capacitances and resistances seen from the capacitors' terminals.

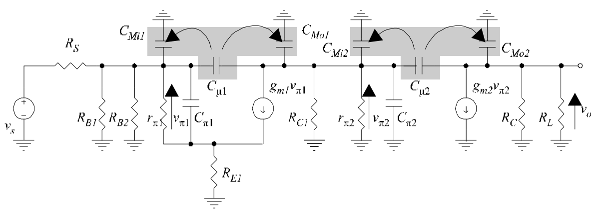

Fig. 11. Small-signal diagram for calculating a high 3-dB frequency.

Using Miller's theorem, a parasitic "floating" capacitance C_μ of transistor can be transferred to two "grounded" capacitances C_{Mi} and C_{Mo}:

$$C_{Mi1} = C_{\mu 1}\left(1 + \frac{r_{\pi 1} g_{m1} (R_{C1} \parallel r_{\pi 2})}{r_{\pi 1} + R_{E1}(\beta_1 + 1)}\right) = = C_{\mu 1}\left(1 + \frac{R_{C1} \parallel r_{\pi 2}}{r_{e1} + R_{E1}}\right) \tag{11}$$

$$C_{Mo1} = C_{\mu 1}\left(1 + \frac{r_{e1} + R_{E1}}{R_{C1} \parallel r_{\pi 2}}\right) \tag{12}$$

$$C_{Mi2} = C_{\mu 2}(1 + g_{m2} R_{C2} \parallel R_L) \tag{13}$$

$$C_{Mo2} = C_{\mu 2}\left(1 + \frac{1}{g_{m2} R_{C2} \| R_L}\right) \tag{14}$$

Using equations (11)-(14), the time constants related to particular capacitances can be easily derived:

$$\tau_{H1} = C_{Mi1} \cdot \left(R_{B1} \| R_{B2} \parallel R_S \parallel (r_{e1} + R_{E1})(\beta_1 + 1)\right) \tag{15}$$

$$\tau_{H2} = C_{\pi 1}\left(r_{\pi 1} \left\| \frac{R_S \| R_{B1} \| R_{B2} + R_{E1}}{1 + g_{m1} R_{E1}} \right.\right) \tag{16}$$

$$\tau_{H3} = \left(C_{Mo1} + C_{Mi2} + C_{\pi 2}\right) \cdot \left(R_{C1} \| r_{\pi 2}\right) \tag{17}$$

$$\tau_{H4} = C_{Mo2} \cdot \left(R_{C2} \| R_L\right) \tag{18}$$

Approximate value of a high 3-dB frequency can be expressed by the formula:

$$f_{H3dB} \approx \frac{1}{2\pi \cdot (\tau_{H1} + \tau_{H2} + \tau_{H3} + \tau_{H4})} \tag{19}$$

or by more precise formula:

$$\boxed{f_{H3dB} \approx \frac{1}{2\pi \cdot \sqrt{\tau_{H1}^2 + \tau_{H2}^2 + \tau_{H3}^2 + \tau_{H4}^2}}} \tag{20}$$

Analysis for low frequencies (i.e., near a low 3-dB frequency)

A small-signal diagram of the amplifier of Fig. 9 is depicted in Fig. 12. The diagram is created assuming that the amplifier operates in low frequency range, i.e., near low 3-dB frequency. The low 3-dB frequency can be derived using a time-constant method. Contrary to the previous analysis, the time constants result from coupling capacitances and resistances seen from the capacitors terminals (when calculating the time constant for a particular capacitance, the remaining capacitances should be treated as a short-circuit). Parasitic capacitances are treated as open-circuits.

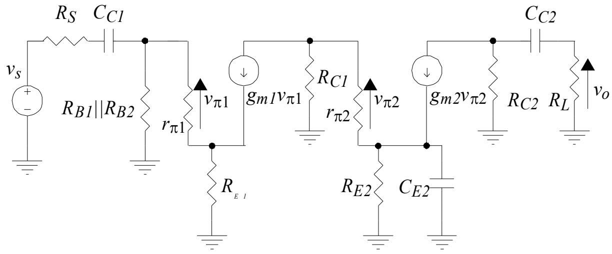

Fig. 12. Small-signal diagram of an amplifier for low frequencies.

The particular time constants are:

$$\tau_{L1} = C_{C1}\left(R_S + R_{B1} \| R_{B2} \| \left((\beta_1 + 1) R_{E1} + r_{\pi 1}\right)\right) \tag{21}$$

$$\tau_{L2} = C_{E2}\left(R_{E2} \left\| \frac{r_{\pi 2} + R_{C1}}{\beta_2 + 1} \right.\right) \tag{22}$$

$$\tau_{L3} = C_{C2}\left(R_{C2} + R_L\right) \tag{23}$$

Approximate value of a low 3-dB frequency can be expressed by the formula:

$$f_{L3dB} \approx \frac{1}{2\pi}\left(\frac{1}{\tau_{L1}} + \frac{1}{\tau_{L2}} + \frac{1}{\tau_{L3}}\right) \tag{24}$$

or by more precise formula:

$$\boxed{f_{L3dB} \approx \frac{1}{2\pi}\sqrt{\frac{1}{\tau_{L1}^2} + \frac{1}{\tau_{L2}^2} + \frac{1}{\tau_{L3}^2}}} \tag{25}$$

4.4. Small-signal analysis of two-stage amplifier with closed feedback loop (configurations B and C).

The main amplifier (from Fig. 9) with closed feedback loop is depicted in Fig. 13. The elements C_F and R_F are added to the main amplifier. Now, the resistor R_{E1} and the additional elements C_F, R_F form the two-port feedback network. The feedback loop passes only AC signals, because the capacitor C_F blocks a DC signal. The capacitance C_F is large enough to be omitted in further small-signal analysis (i.e., C_F can be treated as a short-circuit).

The points (cross symbols) of interrupting the feedback loop in order to extract the main amplifier A and a feedback network B are marked in Fig. 13.

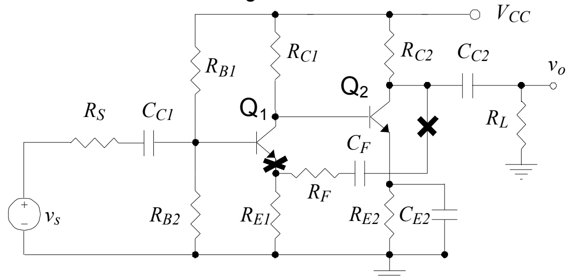

Fig. 13. Diagram of an amplifier with the feedback loop. On the picture are marked spots interrupt the feedback loop to extract the main amplifier A and a feedback network B.

Assuming the operations for middle frequencies (then the coupling capacitors C_{C1}, C_{E2}, C_{C2} and C_F are short-circuits), the diagram in Fig. 13 can be simplified as shown in Fig. 14.

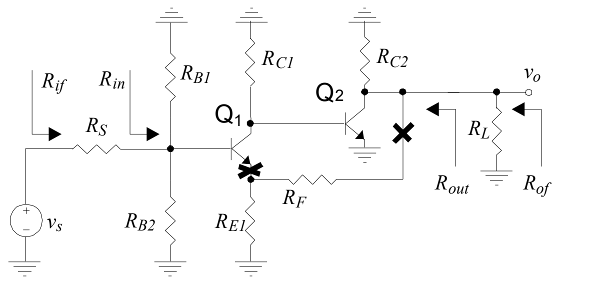

Fig. 14. Diagram of an amplifier with the feedback loop for middle frequencies.

The circuits A and B are shown in Fig. 15 and Fig. 16, respectively.

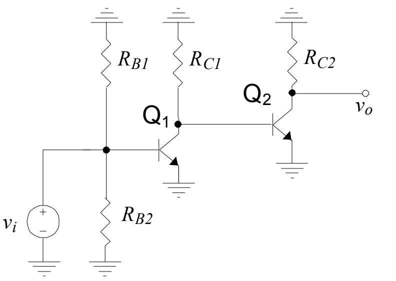

Fig. 15. The main amplifier – circuit A.

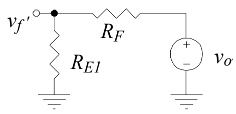

Fig. 16. Two-port feedback network – circuit B.

$$B = \frac{v'_f}{v'_o} = \frac{R_{E1}}{R_{E1} + R_F} \tag{26}$$

$$R_1 = h_{11} = R_{E1} \| R_F \tag{27}$$

$$R_2 = \frac{1}{h_{22}} = R_{E1} + R_F \tag{28}$$

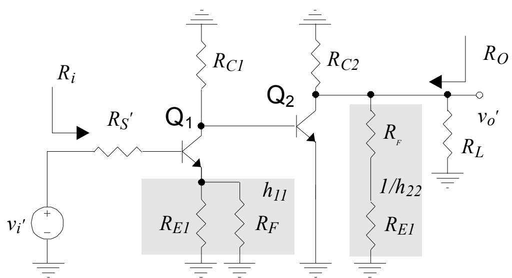

Fig. 17. The modified circuit A'.

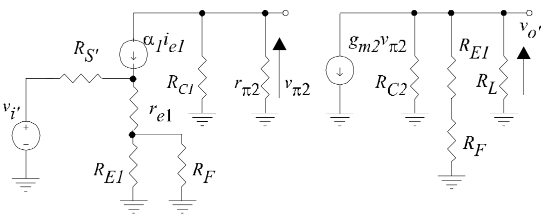

Fig. 18. Small-signal model of circuit A'.

Because the resistors R_{B1} and R_{B2} are not a part of the feedback loop, these elements can be included in an equivalent resistance of Thevenin source:

$$R_{S'} = R_{B1} \| R_{B2} \| R_S \tag{29}$$

The gain of the modified circuit A' is expressed by the formula:

$$\boxed{A' = \frac{v_{o'}}{v_{i'}} = g_{m2}\left(R_{C2}\|R_L\|(R_{E1}+R_F)\right)\frac{\alpha_1(R_{C1} \| r_{\pi 2})}{r_{e1} + R_{E1}\|R_F} \cdot \frac{(r_{e1}+R_{E1}\|R_F)(\beta_1+1)}{(r_{e1}+R_{E1}\|R_F)(\beta_1+1)+R_{S'}}} \tag{30}$$

Hence, the gain of the circuit with closed feedback loop is:

$$\boxed{A_f = \frac{v_o}{v_s} = \frac{A'}{1 + A'B}} \tag{31}$$

The input resistance of the modified circuit A' is:

$$R_i = R_{S'} + (\beta_1 + 1)(r_{e1} + R_{E1}\|R_F) \tag{32}$$

The input resistance of the circuit with closed feedback loop is:

$$R_{if} = R_i(1 + A'B) \tag{33}$$

The input resistance of the real circuit with the feedback is:

$$R_{in} = (R_{if} - R_{S'})\|R_{B1}\|R_{B2} \tag{34}$$

The output resistance of the modified circuit A' is:

$$R_O = R_{C2}\|R_L\|(R_{E1} + R_F) \tag{35}$$

The output resistance of the circuit with feedback is:

$$R_{of} = \frac{R_O}{(1 + A'B)} \tag{36}$$

The output resistance of the real circuit with the feedback is:

$$\frac{1}{R_{of}} = \frac{1}{R_{out}} + \frac{1}{R_L} \quad \Rightarrow \quad R_{out} = \frac{R_{of} R_L}{R_L - R_{of}} \tag{37}$$

5. Measurement of the input resistance

The input resistance is measured using an additional resistor R_S' connected in series with the internal resistance R_S of the generator. The resistor R_s' is "activated" by a button on the front panel (the button is marked as $\boxed{R_{in}}$). During normal operation, the button is not pressed and the resistor Rs' is short-circuited (i.e., the R_s' is "inactive"). After pressing the button, the resistor R_S' is connected in series with R_S, which reduces the amplifier's gain.

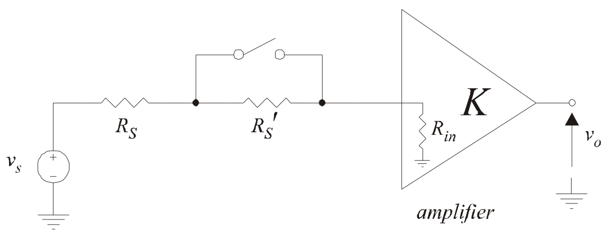

Fig. 19. The method of measuring the input resistance of the amplifier.

Let's denote the output voltage, when R_S' is short circuited (the button is not pressed), as $\boldsymbol{v_o}$. Furthermore, let's denote the output voltage, when R_S' is not short circuited (the button is pressed), as $\boldsymbol{v_o'}$. We can write the following equations:

$$v_0 = K \cdot \frac{R_{in}}{R_{in} + R_S} \cdot v_s \tag{38}$$

$$v_0' = K \cdot \frac{R_{in}}{R_{in} + R_S + R_S'} \cdot v_s \tag{39}$$

$$\frac{v_0}{v_0'} = \frac{R_{in} + R_S + R_S'}{R_{in} + R_S} \tag{40}$$

$$\boxed{R_{in} = \frac{v_0'}{v_0 - v_0'} \cdot R_S' - R_S} \tag{41}$$

6. Measurement of the output resistance

The output resistance is measured using an additional resistor R_L' connected in parallel with the load resistance R_L of the amplifier. The resistor R_L' is "activated" by a button on the front panel (the button is marked as $\boxed{R_{out}}$). During normal operation, the button is not pressed and the resistor R_L' is unconnected (i.e., the R_L' is "inactive"). After pressing the button, the resistor R_L' is connected in parallel with R_L, which reduces the amplifier gain.

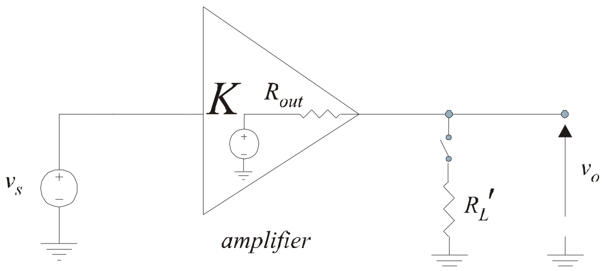

Fig. 20. The method of measuring the output resistance of the amplifier.

Let's denote the output voltage, when R_L' is unconnected (the button is not pressed), as v_o. Furthermore, let's denote the output voltage, when R_L' is connected (the button is pressed), as v_o'. We can write the following equations:

$$v_0 = K \cdot \frac{R_L}{R_L + R_{out}} \cdot v_s \tag{42}$$

$$v_0' = K \cdot \frac{R_L \parallel R_L'}{R_L \parallel R_L' + R_{out}} \cdot v_s \tag{43}$$

$$\frac{v_0'(R_L \parallel R_L' + R_{out})}{R_L \parallel R_L'} = \frac{v_0(R_L + R_{out})}{R_L} \tag{4}$$

$$\boxed{R_{out} = \frac{R_L \cdot R_L'}{\dfrac{R_L \cdot v_0'}{v_0 - v_0'} - R_L'}} \tag{44}$$

7. Elements parameters

Parameter	Value
β for Q_1 and Q_2	200
C_μ for Q_1 and Q_2	4.5 pF
f_T for Q_1 and Q_2	150 MHz
V_{CC}	+12 V
R_{B1}	51 kΩ
R_{B2}	6.2 kΩ
R_{C1}	8.2 kΩ
R_{E1}	560 Ω
R_{C2}	2.2 kΩ
R_{E2}	1.2 kΩ
R_L	5.1 kΩ
R_{FB}	18 kΩ
R_{FC}	36 kΩ
$R_{S'}$	2 kΩ
$R_{L'}$, R_S	1 kΩ
C_{C1}, C_{C2}	6.8 μF
C_{E2}	680 nF

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