Resonant amplifier

3

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1. Introduction

This lab allows you to explore the basic characteristics of the resonant amplifier. In this lab you will measure three resonant amplifiers: with a single, double and triple parallel (current) resonance circuit. **Prior to the lab, you need to have some theoretical background (basic information has been presented in the text below). Teacher is obliged to check your preparation for this lab.**

2. Measurements

Remark: all the amplifiers are tuned at the mid-band frequency f_0 = 465 kHz.

2.1 Measuring the frequency response of the amplifier with a single parallel resonance circuit

The measurements should be performed for the following circuits:

a) the circuit without resistive load: the output at the transistor collector (WY1, P1-open, P2-open);
b) the circuit with the load resistor R_0 = 18 kHz connected via a coupling capacitance to the transistor collector: the output at the transistor collector (WY1, P1-shorted, P2-open);
c) the circuit without resistive load: the output from the capacitance divider (WY2, P1-open, P2-open);
d) the circuit with the load resistor R_0 = 4,3 kHz connected via a coupling capacitor to the capacitance divider: the output from the capacitance divider (WY2, P1-open, P2-shorted);

Measurements are made by hand, setting up the frequency and reading out the output voltage. Set the input voltage V_{we} = 20 mV$_{rms}$ on the voltage generator. The frequency of the generator to be varied in such a way that the following quantity could be read out precisely: the resonant frequency (f_0), the gain (amplification) at the resonance frequency $A_0 = K_u(f_0)$ as well as the 3-dB low (f_{L3dB}) and 3-dB high (f_{H3dB}) frequencies and the 3-dB bandwidth, where $K_u = V_{wy} / V_{we}$. The frequency characteristics should be measured for the following frequencies:

f_{we} [kHz]	f_{L10dB}	f_{L6dB}	f_{L3dB}	f_0	f_{H3dB}	f_{H6dB}	f_{H10dB}
K_u [V/V]	$0.316 \cdot A_0$	$0.5 \cdot A_0$	$0.707 \cdot A_0$	A_0	$0.707 \cdot A_0$	$0.5 \cdot A_0$	$0.316 \cdot A_0$

Millivoltmeter used to measure the output voltage of the amplifier should have a large input resistance and low input capacitance. Therefore, it is equipped with a probe (note: the probe introduces the constant signal attenuation 1:1700, which should be included in the measurements).

2.2 Measuring the frequency response of the amplifiers with double and triple parallel resonance circuits

2.2.1. Observe characteristics of the amplifier with a double resonant circuit for the following values of coupling capacitances:

a) C_{12a}=68pF (position 1 of the selector switch P1);
b) C_{12b}=47pF (position 2 of the selector switch P1);
c) C_{12c}=33pF (position 3 of the selector switch P1);
d) C_{12d}=18pF (position 4 of the selector switch P1).

Carry out the measurements for C_{12a} = 68pF and C_{12c} = 33pF. The measurements are made semi-automatically. Select SWEEP mode on the generator, and switch the oscilloscope to the mode X-Y (Fig.1). After pressing the SWP, an auxiliary saw-tooth low-frequency generator turns on, whereas signal of the generator controls the frequency of the main generator. In this way, the frequency of the main generator is swept to the extent determined by the main handwheel of the generator. Connect the control signal (sweeps) to the input X of the oscilloscope and then make a joint between the output of the test circuit and input Y of the oscilloscope. Observe on the oscilloscope a picture of the frequency response of the amplifier (reflected symmetrically with respect to the axis X). To read the coordinates of specific points on the displayed characteristic, turn off momentarily sweep (SWP), find the specific point and read the frequency on the frequency meter and then the amplitude of the output voltage on the voltmeter.

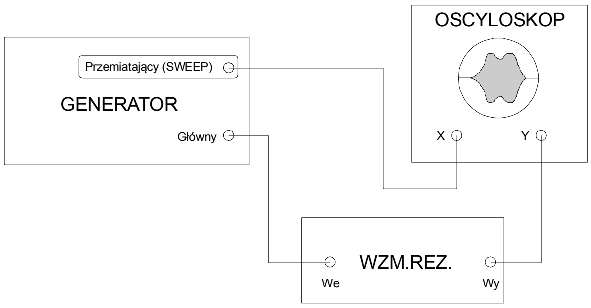

Fig.1. Amplifier with a single parallel resonance circuit - measurement arrangement.

Set the input voltage V_{we} = 100 mV$_{rms}$ on the voltage generator.
Redraw the results out of the oscilloscope; write down the coordinates of the most important points of the characteristic in the table.

2.2.2. Observe characteristics of the amplifier with a triple resonant circuit for the following pairs of coupling capacitances:

a) C_{12a}=C_{23a}=300pF (position 1 of the selector switch P2);
b) C_{12B}=C_{23b}=270pF (position 2 of the selector switch P2);
c) C_{12C}=C_{23c}=250pF (position 3 of the selector switch P2);

Carry out measurements for C_{12a} = 300pF. The measurements are made semi-automatically (see section 2.2.1). Set the input voltage V_{we} = 100 mV$_{rms}$ on the voltage generator. Redraw the results out of the oscilloscope; write down the coordinates of the most important points of the characteristic in the table.

3. Elaboration of the results

3.1 Based on the formulas given in the measurement protocol and measurements of f_0, calculate the voltage gain $A_0 = Ku(f_0)$ and 3dB-bandwidth Δf_{3dB} for the measurements from the point 2.1. The value of inductance L is determined on the basis of the measurements of f_0 and values of capacitances in the resonant circuit. Place the results of auxiliary calculations in the measurement protocol table. Assume the following parameter values in the calculations:

g_m = 50 mS (the value read out from the characteristics of the transistor BF215 for I_C = 1.5 mA)

Q_L = 50; r_0 = 250 kΩ; $C_0 \approx 0$.

The results of the calculations and measurements put in a common table in the protocol, so they can be easily compared.

3.2 Using the results from the point 2.1 a, b, c, d, plot the frequency responses $20 log|K_u| = f(f)$ for each amplifier together on a single chart. Use the following settings for the axes: X = logarithmic, Y = linear.

3.3 For the other circuits, draw the measured frequency characteristics $20log|K_u|=f(f)$.
3.4 Draw your own conclusions. Compare the circuits between themselves and comment on the agreement between the theoretical calculations with the measurements.

4. Theory

Resonant LC amplifiers, named so because of the use of resonant LC circuits as a load of the transistor, are narrow-pass-band amplifiers. The magnitude of their gain characteristics are similar to the characteristics of the active RC pass-band filters, however, they are applied for larger frequencies than active RC filters.
Resonant LC amplifiers are mainly used in radio communication devices as:

- high-frequency amplifiers having a relatively wide bands, for example as a antenna amplifiers;
- intermediate frequency amplifiers, for example, in receivers with a double-conversion - in this case the amplifiers determine the selectivity of the receiver and they are characterized by a narrow bandwidth and steep slopes of the frequency characteristics.

Resonant LC amplifiers are used wherever it is necessary filtering and amplification of signals, and it is impossible for technical or economic use of other solutions.

4.1 Amplifier with bipolar transistor and a single resonant LC circuit.
The simplest resonant amplifier consists of a transistor connected with a parallel resonant circuit (Fig. 2).

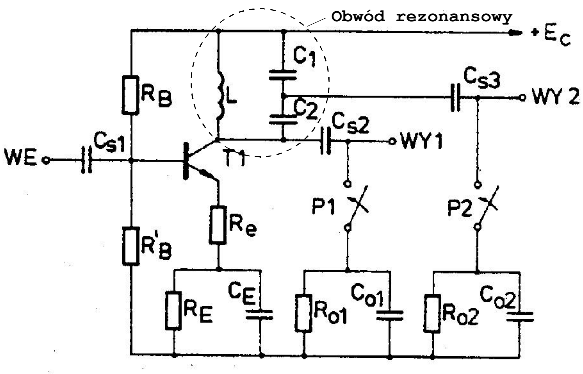

Fig.2. Amplifier with a single parallel resonance circuit.

Small-signal equivalent circuit of the amplifier of Fig. 2 is shown in Fig. 3.

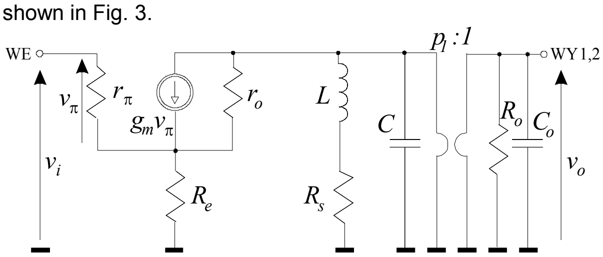

Fig.3. Small-signal equivalent circuit of the amplifier of Fig. 2

where
L – inductance of the coil,
R_s – winding resistance,
C – capacitance of the resonant circuit

$$C = \frac{C_1 C_2}{C_1 + C_1} \tag{1}$$

p_1 – transformation ratio resulting from the use of the split capacitance (the split capacitance has properties of a split inductance only for high values of the resistive load, which is true for the circuit used in the exercise). $p_1 = \frac{C_1 + C_2}{C_1} = \frac{C_2}{C}$ if the load is connected to the split capacitance, and p_1 = 1 if the load is connected directly to the collector of the transistor (the transformation is used to reduce the impact of the load resistance on the resonant circuit).

R_0, C_0 – resistance and capacitance of the load.
The small-signal equivalent circuit does not include the internal capacitance of the transistor. In this case, they do not have a significant effect on the amplifier in the vicinity of the resonant frequency. The coupling capacitance C_{S1}, C_{S2} and C_{S3} are short-circuited, due to their large numerical values.
The equivalent circuit of Fig. 3 can be further simplified by introducing the dynamic resistance of the coil instead of the winding resistance, as shown in Fig. 4.

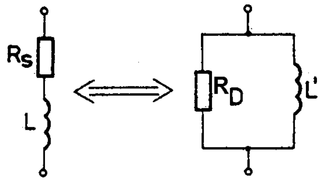

Fig.4. Equivalence between the winding resistance and the dynamic resistance of the coil.

Comparing admittances of the two circuits, we have

$$R_D = R_S\left(1+Q_L\right)^2 \quad \text{i} \quad L' = L\left(1+\frac{1}{Q_L^2}\right) \tag{2}$$

where
R_S – winding resistance of the coil,
R_D – dynamic resistance of the coil,
Q_L – quality factor of the coil, $Q_L = \frac{\omega \cdot L}{R_S}$.
In practice Q_L»1, thus it can be simplified as follows:

$$R_D \cong R_S Q_L^2 \ , \quad L' \cong L\ . \tag{3}$$

The applied feedback (the resistance R_e connected to the emitter of the transistor) results in the following features of the circuit, as compared to the circuit with the grounded emitter:
- the resistance r_0 seen by the resonant circuit is F-fold greater, where

$$F = 1 + g_m R_e \tag{4}$$

- transconductance is reduced to g_m*, where:

$$g_m^* = \frac{g_m}{1+g_m R_e}\ . \tag{4}$$

As a result, the frequency parameters of the amplifier and its gain depend essentially only on the parameters of the resonant circuit.
Small-signal equivalent circuit of the amplifier in which the above conclusions are taken into account is presented in Fig. 5. In Fig. 5, the load is transformed into the resonant circuit.

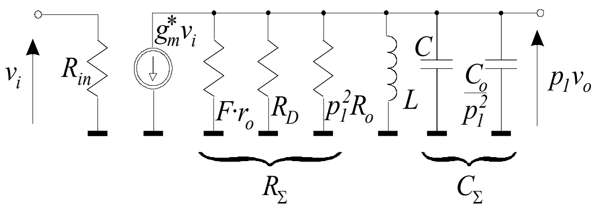

Fig. 5. Equivalent circuit of the amplifier with a single resonant circuit, taking into account certain parameters of the transistor, the dynamic resistance of the coil, and the load.

Denoting:

$$R_\Sigma = \left(F \cdot r_o\right) \Big\| R_D \Big\| \left(p_1^2 R_o\right), \tag{5}$$

$$C_\Sigma = C + \frac{C_o}{p_1^2}\ , \tag{6}$$

we have:

$$p_1 v_o = -\frac{g_m^* \cdot v_i}{\frac{1}{R_\Sigma} + sC_\Sigma + \frac{1}{sL}}\ . \tag{7}$$

After some mathematical manipulations, one can obtain the following formula for the voltage gain of the circuit:

$$K_v = \frac{v_o}{v_i} = -g_m^* \frac{R_\Sigma}{p_1} \cdot \frac{\frac{s}{R_\Sigma C_\Sigma}}{s^2 + \frac{s}{R_\Sigma C_\Sigma} + \frac{1}{LC_\Sigma}} \quad (8)$$

Denoting:

$$K_{vr} = -\frac{g_m^* R_\Sigma}{p_1} \quad (9)$$

$$\beta_{3dB} = \frac{\omega_o}{Q} = \frac{1}{C_\Sigma R_\Sigma} \quad (10)$$

$$\omega_o = \frac{1}{\sqrt{LC_\Sigma}} \quad (11)$$

we can thus write,

$$K_v = K_{vr} \cdot \frac{\frac{\omega_o}{Q} s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}, \quad (12)$$

where:

K_{vr} – gain at the resonant frequency;
ω_o – resonant angular frequency;
β_{3dB} – 3-dB bandwidth;
Q - quality factor of the amplifier.

The parameter Q can be given in the following form:

$$Q = \omega_o C_\Sigma R_\Sigma = R_\Sigma \sqrt{\frac{C_\Sigma}{L}} \quad (13)$$

One can see that the form of equation (14) is analogous to the transmittance of the active second-order pass-band RC filter. Now, let us consider the distribution of poles of the function describing the impedance of the circuit that loads the transistor:

$$Z(s) = \frac{1}{\frac{1}{R_\Sigma} + sC_\Sigma + \frac{1}{sL}} = \frac{s}{s^2 C_\Sigma + \frac{s}{R_\Sigma} + \frac{1}{L}} \quad (14)$$

The function $Z(s)$ has one zero at point $s_0 = 0$ and two complex conjugate poles:

$$s_1, s_1^* = -\frac{1}{2R_\Sigma C_\Sigma} \pm j\sqrt{\frac{1}{LC_\Sigma} - \left(\frac{1}{2R_\Sigma C_\Sigma}\right)^2} \quad (15)$$

We can write $\frac{1}{2R_\Sigma C_\Sigma} = \frac{\omega_o}{2Q}$ because $Q = R_\Sigma C_\Sigma \omega_o = \frac{R_\Sigma}{\omega_o L}$

and $\omega_o = \frac{1}{\sqrt{LC_\Sigma}}$.

Therefore, the poles can be represented as:

$$s_1, s_1^* = -\frac{\omega_o}{2Q} \pm j\sqrt{\omega_o^2 - \left(\frac{\omega_o}{2Q}\right)} \quad (16)$$

If the quality factor Q of the circuit is large enough, then

$$\omega_o^2 >> \frac{\omega_o^2}{(2Q)^2} \quad (17)$$

and the result is:

$$s_1, s_2^* \approx -\frac{\omega_o}{2Q} \pm j\omega_o . \quad (18)$$

Distribution of zeros and poles is shown in Fig.6.

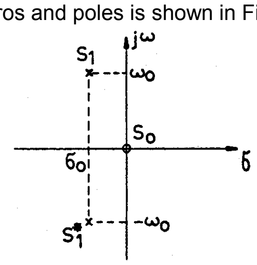

Fig. 6. Distribution of zeros and poles of the impedance Z(s).

Real part of the poles s_1 i s_1^*, denoted by σ_0, is

$$\sigma_o = -\frac{\omega_o}{2Q} \quad (19)$$

The parameter σ_0 is called the damping factor, whereas the imaginary part of (17) is called the tuned angular frequency. The function $Z(s)$ can be also expressed as:

$$Z(s) = \frac{1}{C_\Sigma\left(s^2 + s\frac{1}{R_\Sigma C_\Sigma} + \frac{1}{LC_\Sigma}\right)} = \frac{1}{C_\Sigma} \cdot \frac{s - s_o}{(s - s_1)(s - s_1^*)} \quad (20)$$

However, in the present case $s_0 = 0$.
Factors $(s - s_0)$, $(s - s_1)$, $(s - s_1^*)$ can be treated as vectors. These vectors for arbitrarily chosen value of $s = j\omega$ is shown in Fig. 7.

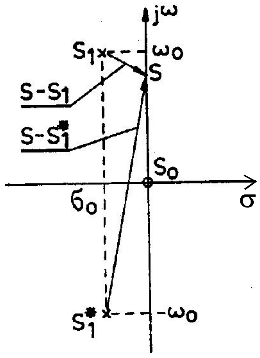

Fig. 7. Location of the vectors: $|s - s_0|$, $|s - s_1|$, $|s - s_1^*|$ as a function of $s = j\omega$ for the impedance $Z(s)$.

For the case of relatively narrow band ($\sigma \ll \omega_o$), we are interested only in the values of s that are placed in the neighborhood of s_1. Then, the ratio

$$\frac{|s - s_o|}{|s - s_1^*|} \approx \frac{1}{2} . \quad (21)$$

This is seen in Fig. 7. Equation (22) can written as

$$Z(s) = \frac{1}{2C_\Sigma |s - s_1|} . \quad (22)$$

The simplification (22) leads to the so-called narrowband approximation. As a consequence of the simplification, we take into account only the positive part of the imaginary axis. In addition, the damping factor is always negative, that is, we are interested in only one quadrant of the coordinate system (σ, ω). Because it is primarily important for us variation of the function $Z(s)$ along the imaginary axis, it is convenient to rotate the coordinate system of 90° in a clockwise direction as shown in Fig. 8.

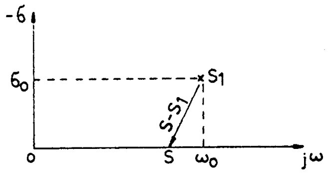

Fig. 8. Rotated coordinate system.

Vector | s-s1 | is a vector whose amplitude (length) varies as a function of frequency. Measuring the length of this vector, one can determine variation of $Z(s)$. The maximum value of $|Z(s)|$ occurs when $|s - s_1| = r_1$ reaches the minimum value, that is at the resonance (ω_0) - or $Z(j\omega_0)$. We can calculate a relative change of $|Z(s)|$ with respect to the maximum value of $|Z(j\omega_0)|$. Let us define the relative change of $|Z(s)|$ at $s = j\omega_1$.

$$|Z(j\omega_1)| = \frac{1}{2C_\Sigma}\left|\frac{1}{j\omega_1 - s_1}\right| \quad (23)$$

$$|Z(j\omega_o)| = \frac{1}{2C_\Sigma}\left|\frac{1}{j\omega_o - s_1}\right| \quad (24)$$

Dividing equation (24) by (25), we get:

$$A(j\omega_1) = \left|\frac{Z(j\omega_1)}{Z(j\omega_o)}\right| = \left|\frac{j\omega_o - s_1}{j\omega_1 - s_1}\right| \quad (25)$$

Let us find ω_1 for which the vector $|s - s_1|$ forms an angle of 45° with a line which is perpendicular to the frequency axis and passes through the pole s_1, see Fig. 9).

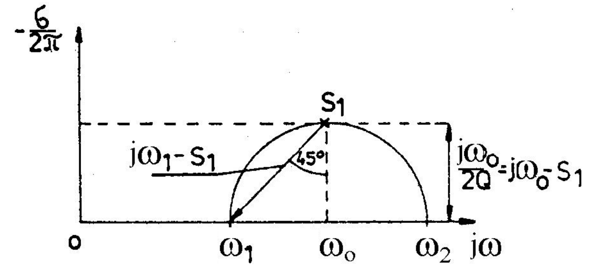

Fig. 9. Determination of the 3-dB amplifier bandwidth.

From the fig. 9, we define

$$A(j\omega_1)=\left|\frac{j\omega_o - s_1}{j\omega_1 - s_1}\right| = \frac{1}{\sqrt{2}} = 0.707 \qquad (26)$$

One can observe that at ω_2 (see Fig. 9), the function $A(j\omega)$ takes the same value as in (27). In this way, we can calculate the 3-dB bandwidth as follows:

$$\Delta\omega_{3dB} = \omega_2 - \omega_1 \qquad (27)$$

In the case of parallel resonance circuit, the pole s_1 is on the top of a semi-circle with a diameter of $\Delta\omega_{3dB}$ and the center at ω_o. It can also be seen from Fig. 9 that the diameter of the semi-circle is equal to twice the value of the damping factor, leading to the relationship:

$$\Delta\omega_{3dB} = \frac{\omega_o}{Q}\text{, where } Q = \frac{\omega_o}{\Delta\omega_{3dB}} \qquad (28)$$

Besides, there is a relationship between ω_1 and ω_2:

$$\omega_o = \frac{\omega_1 + \omega_2}{2} \qquad (29)$$

The impedance of the circuit at resonance is:

$$\left|Z(j\omega_o)\right| = \left|\frac{1}{2C\Sigma\sigma_o}\right| = R_\Sigma \qquad (30)$$

Following as before, we can obtain the resonance curve as shown in Fig. 10.

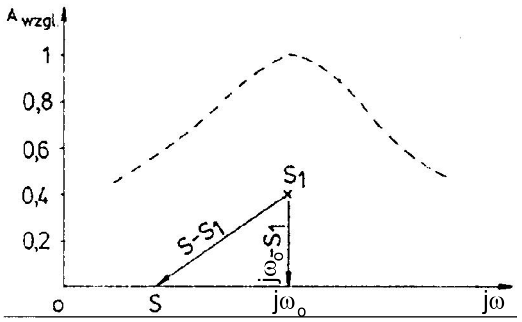

Fig. 10. Geometrical determination of the resonance curve of the amplifier with a single resonant circuit.

4.2 Amplifier with a pair of resonant circuits coupled capacitively

Fig. 11 shows an amplifier with a pair of LC resonant circuits.

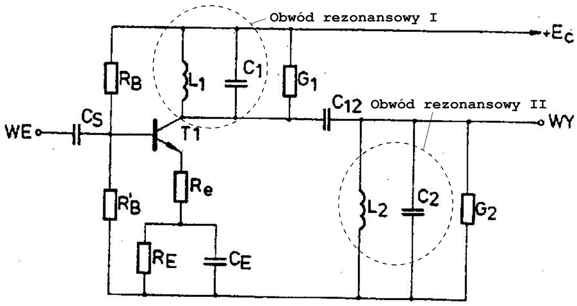

Fig. 11. Amplifier with a pair of LC resonant circuits

Analysis of this amplifier is more complicated than the previous one and is beyond the scope of this laboratory. The result of such an analysis (carried out by the graphical method) is the characteristic shown in Fig. 12. The Coupling coefficient ℵ is determined by the formula: $\aleph = \frac{C_{12}}{\sqrt{C_{11}C_{22}}}$, where $C_{11}=C_1+C_{12}$, $C_{22}=C_2+C_{12}$.

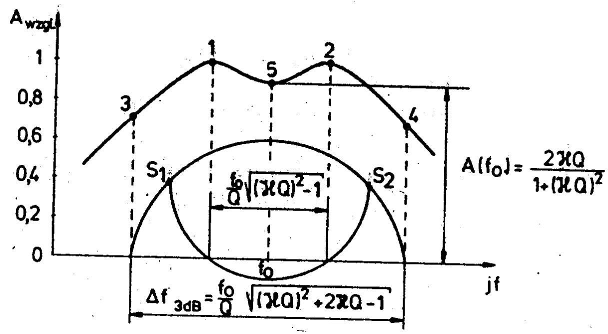

Fig. 12. Geometrical determination of the frequency response of an amplifier with a pair of resonant circuits coupled capacitively.

Fig. 12 shows the frequency response for the specific values of the elements. Depending on relations between capacitances, these type characteristics take different shapes. In Fig. 13, an exemplary family of universal curves is shown.

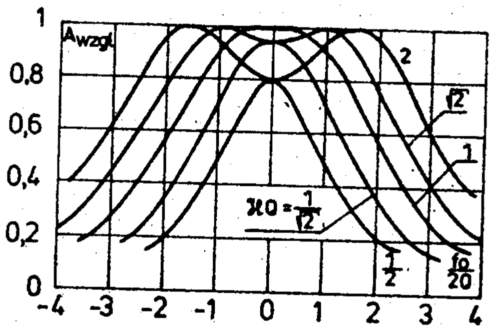

Fig. 13. Family of universal curves for an amplifier with a pair of resonant circuits coupled capacitively.

These curves summarize the properties of the circuit incorporated in the case where the quality factors of the individual circuits are equal $Q_1 = Q_2 = Q$. In the figure, the abscissa is scaled in units of $f_0 / 2Q$, and the ordinate - in the relative amplitudes. Analyzing the shape of these curves, it can be concluded that an increase in the degree of the coupling causes an increase in the distance between the vertices of the curves, however the gain corresponding to the vertices remains constant.

4.3 Amplifier with a triple resonant circuit coupled capacitively

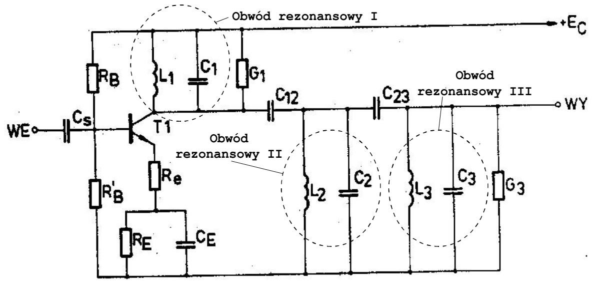

Fig. 14. Amplifier with a triple resonant circuit coupled capacitively

Fig. 14 shows an amplifier with a triple resonant circuit coupled capacitively. As in the case of an amplifier with a pair of LC resonant circuits, there will be no analysis performed here. The frequency response of the amplifier is even more complicated, see Fig. 15. A universal characteristic is shown in Fig. 16.

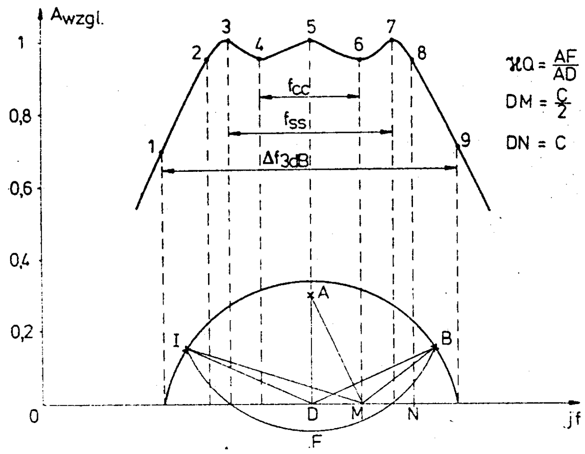

Fig. 15. Geometrical determination of the frequency response of an amplifier with a triple resonant circuit coupled capacitively.

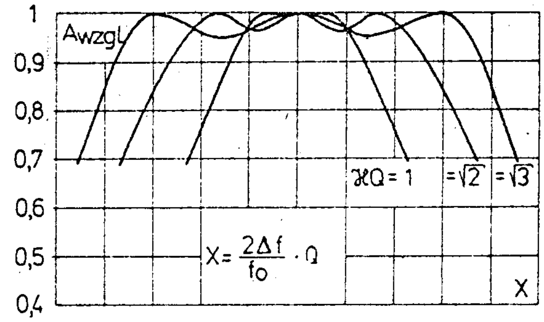

Fig. 16. Family of universal curves for an amplifier with a triple resonant circuit coupled capacitively.

The Coupling coefficient ℵ is determined by the formula:

$$\aleph = \frac{C_{12}}{\sqrt{(C_1 + C_{12})\left(\frac{C_2}{2} + C_{12}\right)}} = \frac{C_{23}}{\sqrt{(C_3 + C_{23})\left(\frac{C_2}{2} + C_{23}\right)}} .$$

If the coupling capacitances satisfy the condition $C_{12} = C_{23}$, then:

$$\aleph = \frac{C_{12}}{\sqrt{\frac{C_{11}C_{22}}{2}}} = \frac{C_{23}}{\sqrt{\frac{C_{33}C_{22}}{2}}} ,$$

where $C_{11}=C_1+C_{12}$, $C_{22}=C_2+C_{12}+C_{23}$, $C_{33}=C_3+C_{23}$.

One can see from Fig. 16 that an increase in the product ℵQ causes a significant bandwidth extension without improve significantly the pits. The analysis of the circuit is made on the assumption that the quality factor of the central resonant circuit Q2 = ∞. The results obtained are already in accordance with the theory when $Q_2 = 10Q_1 = 10Q_3 = 10Q$. In this case, however, an 'attenuation' of the lateral vertices of the characteristic in Fig.17 occurs.

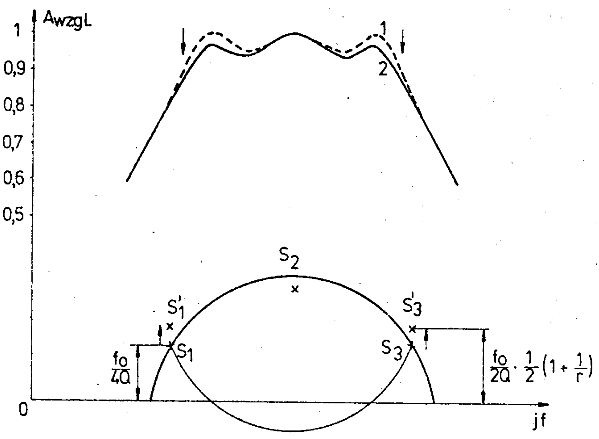

Fig. 17. The shape of the frequency response of the amplifier for $Q_2 < \infty$.

5. Schemes studied circuits

5.1 Amplifier with a single resonant circuit

In the circuit, it can be measured the output signal either directly from the collector of the transistor (not switching the load on - P1 open or switching the load 18kΩ on - P1 closed) or from the capacitive divider (not switching the load on - P2 open, or switching the load 4.3kΩ on - P2 closed).

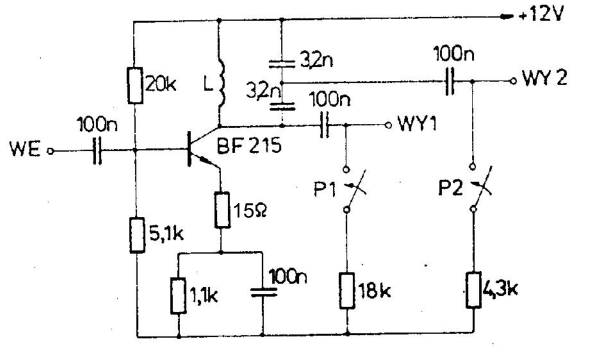

Fig.18. Amplifier with a single parallel resonance circuit.

5.2 Amplifier with a pair of resonant circuits coupled capacitively

In the circuit shown in Fig. 19, you can change the coupling coefficient ℵ by changing the capacitance coupling (rotary switch P1). Quality factors of the circuits I and II are equal, i.e. $Q_1=Q_2=Q$.

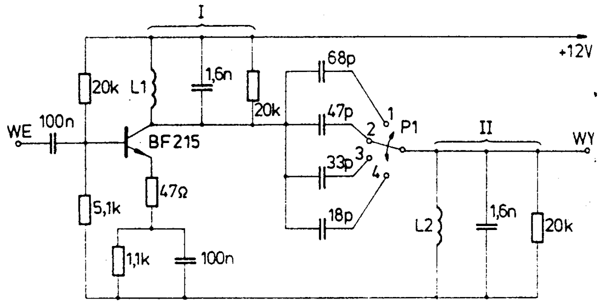

Fig.19. Amplifier with a pair of resonant circuits.

5.3 Amplifier with a triple resonant circuit coupled capacitively

In the circuit of Fig. 20, it is possible to change the coupling coefficient ℵ by simultaneous change of the two coupling capacitances (rotary switch P2). Quality factors of the circuits I and II are equal, i.e. $Q_1 = Q_3 = Q$. In order to satisfy the condition: $Q_2=10Q=10Q_1=10Q_3$, the load resistive should be of 2.2kΩ.

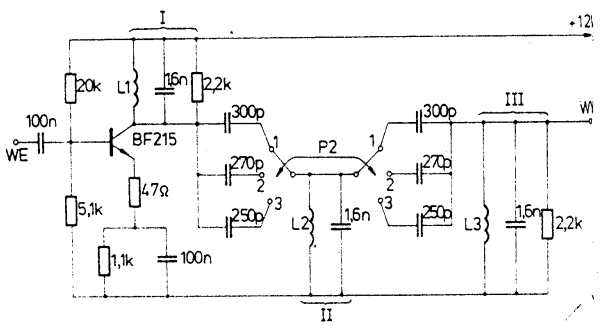

Fig.20. Amplifier with a triple resonant circuit.

References

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