

# Introduction to Basic Electronic Circuits

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## 1. Symbolic used:

- small letter with capital index – actual value of a signal
- small letter with small index – small signal component
- capital letter with capital index – constant component

actual value of a signal = constant component + small signal component

Below example shows transfer characteristics of semiconductor diode with all above types of signal explained.

$$i_D = I_D + i_d \quad v_D = V_D + v_d \quad (1, 2)$$

Current of semiconductor diode  $i_D$  can be described as a function of voltage across the device  $v_D$  by:

$$i_D = I_S \left( e^{\frac{v_D}{mV_T}} - 1 \right) \quad (3)$$

where:  $I_S$  is saturation current (technological parameter given by manufacturer),  $m$  – emission coefficient (equal to 1 for silicon),  $V_T = kT/q$  is thermal voltage,  $k = 1.381e-23[\text{VC/K}]$  is the Boltzman constant,  $T$  is absolute temperature,  $q$  is the electron charge ( $1.602e-19[\text{C}]$ ), for  $T=300[\text{K}]$  thermal potential is equal to  $V_T=25.8\text{mV}$ .

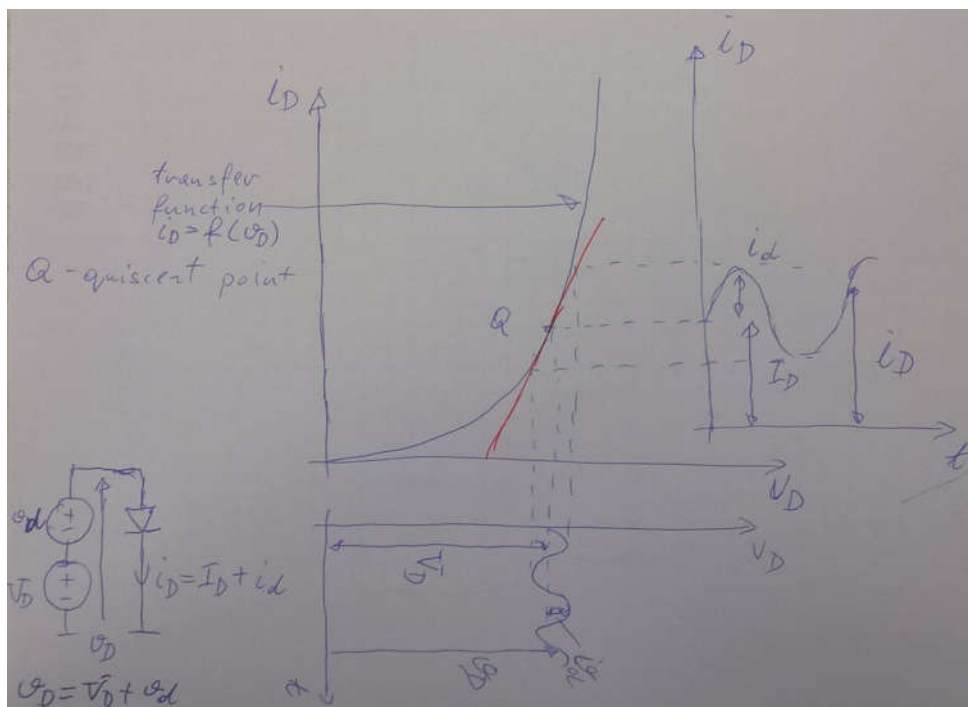


Fig. 1. Explanation of actual value, constant component and small-signal component which flows/exists at the semiconductor diode in forward bias.

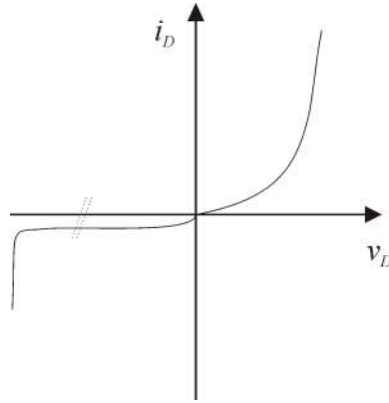


Fig. 2. Voltage – current transfer function of real semiconductor diode.

## 2. What is linear circuit?

It is a such circuit which conforms to superposition principle – what means that the output of the circuit for which is sum of two signals applied is equal to sum of outputs for individually applied inputs as given by equation:

$$H(x_1 + x_2) = H(x_1) + H(x_2) \quad (4)$$

## 3. How to make circuit linear?

- Limit signals processed.
- Introduce negative feedback loop.
- Compose cascaded circuit from nonlinear and inverse nonlinear subcircuits.

This lecture will partially cover two first methods.

## 4. How far signal should be limited to be sure of linear range of operation?

It depends of nonlinearity to be “linearized”. For semiconductor diode described by equation (3) it can be estimated by expansion (3) into Taylor power series by:

$$\begin{aligned} i_D = (I_D + i_d) &= I_S \left( e^{\frac{V_D + v_d}{mV_T}} - 1 \right) = -I_S + I_S e^{\frac{V_D}{mV_T}} e^{\frac{v_d}{mV_T}} \cong I_D e^{\frac{v_d}{mV_T}} = \\ &= I_D \left[ 1 + \frac{v_d}{mV_T} + \frac{1}{2!} \left( \frac{v_d}{mV_T} \right)^2 + \frac{1}{3!} \left( \frac{v_d}{mV_T} \right)^3 + \dots \right] \end{aligned} \quad (5)$$

If we use only 3 first terms of (5) then linearity condition can be estimated as:

$$|v_d| \ll 2mV_T \quad (6)$$

The same linear model as given by (5) can be also obtained by finding the derivative of diode current in quiescent point Q – which is also called working point.

$$\begin{aligned} g_d &= \left. \frac{\partial i_D}{\partial v_D} \right|_{v_D=V_D} = \frac{1}{mV_T} I_S e^{\frac{v_D}{mV_T}} \Big|_{v_D=V_D} = \frac{1}{mV_T} I_S \left( e^{\frac{V_D}{mV_T}} - 1 + 1 \right) = \\ &= \frac{I_D}{mV_T} + \frac{I_S}{mV_T} \end{aligned} \quad (7)$$

where:  $I_D = I_S \left( e^{\frac{V_D}{mV_T}} - 1 \right)$  and assuming that  $V_D \gg mV_T$  (for  $V_D \gg 0.25V$ ) equation (7) can be simplified to the form:

$$g_d = \frac{I_D}{mV_T} \quad (8)$$

Finally small-signal model of semiconductor diode is simple conductance device of value given by (8).

**Example 1.** Please calculate voltages across a semiconductor diode assuming  $V_T=25[mV]$ ,  $I_S=1e-14[A]$ ,  $m=1$  and  $I_D = 1..100[mA]$ . Then calculate small-signal conductance of the diode and at the and calculate small-signal currents in the circuit given in Fig. 3 and estimate if small-signal condition is satisfied.

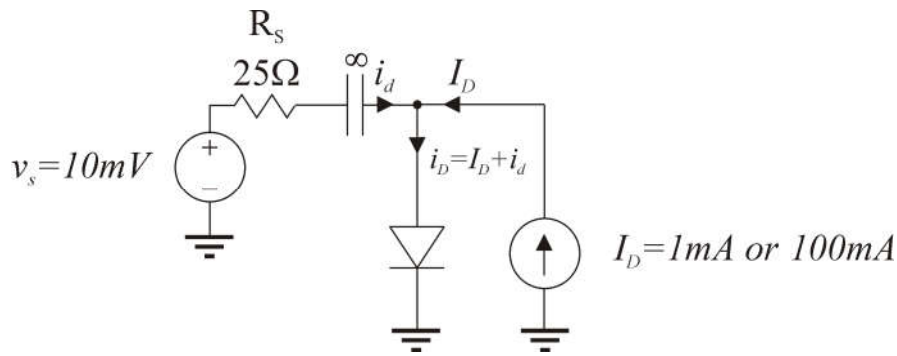


Fig. 3. Semiconductor diode with splitted small-signal and working point supply.

**Solution:** finding  $v_D$  from (3) gives:

$$v_D = mV_T \ln\left(\frac{i_D}{I_S}\right) \quad (9)$$

what for given data results in:

$$V_{D|I_D=1mA} = mV_T \ln\left(\frac{I_D}{I_S}\right) = 25mV \ln\left(\frac{1mA}{1 \cdot 10^{-14} A}\right) = 0.633V$$

and

$$V_{D|I_D=100mA} = mV_T \ln\left(\frac{I_D}{I_S}\right) = 25mV \ln\left(\frac{100mA}{1 \cdot 10^{-14} A}\right) = 0.748V$$

Conductances of equivalent linear circuit:

$$g_d = \frac{I_D}{mV_T} = 0.04S \text{ or equivalent } r_d = \frac{1}{g_d} = 25\Omega \text{ for } I_D=1mA$$

$$g_d = \frac{I_D}{mV_T} = 4S \text{ or equivalent } r_d = \frac{1}{g_d} = 0.25\Omega \text{ for } I_D=100mA$$

Equivalent small-signal model of the circuit from Fig. 3 for finding  $i_d$  component is presented in the Fig. 4. Together with  $R_S$  resistor it compose simple voltage divider thus diode current and voltage across it can be expressed as:

$$i_d = \frac{v_s}{R_s + r_d} = \frac{10mV}{25\Omega + 25\Omega} = 200\mu A$$

$$v_d = v_s \frac{r_d}{R_s + r_d} = 10mV \frac{25\Omega}{25\Omega + 25\Omega} = 5mV \quad \text{for } I_D=1mA \quad \text{and}$$

$$i_d = \frac{v_s}{R_s + r_d} = \frac{10mV}{25\Omega + 0.25\Omega} = 396\mu A$$

$$v_d = v_s \frac{r_d}{R_s + r_d} = 10mV \frac{0.25\Omega}{25\Omega + 0.25\Omega} = 99\mu V \quad \text{for } I_D=100mA \quad \text{and}$$

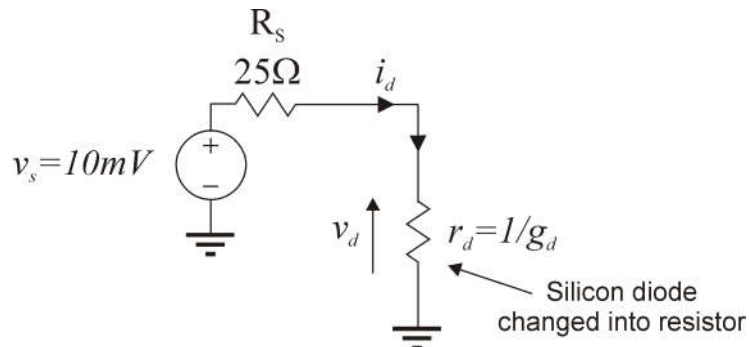


Fig. 4. Equivalent small-signal model of circuit from Fig. 3.

## 5. Bipolar Junction Transistor (BJT) in forward active region

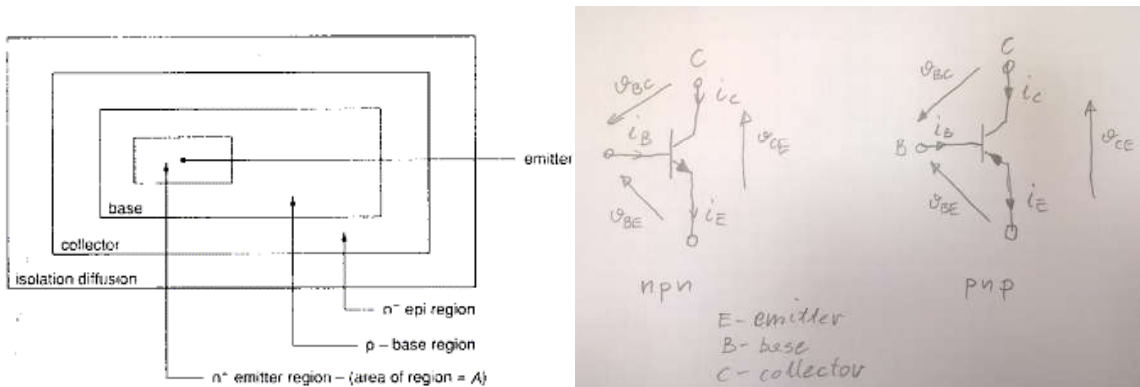


Fig. 5. Internal structure of npn BJT transistor and symbols of npn and pnp devices.

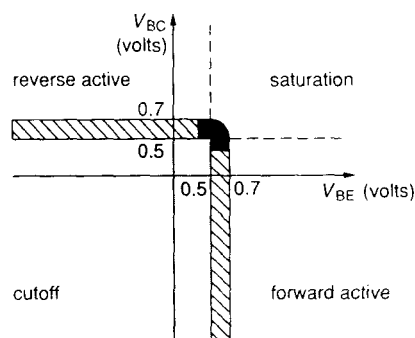


Fig. 6. Operation ranges for npn BJT.

For  $v_{BE} > 0.5V$   $v_{BC} < 0.3V$  npn BJT operates in forward active region and its currents of collector, emitter and base terminals can be expressed as:

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \quad (10)$$

$$i_E = \frac{I_S}{\alpha} e^{\frac{v_{BE}}{V_T}} \quad (11)$$

$$i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} \quad (12)$$

where:  $I_S$  is the saturation current,  $\beta = \frac{\alpha}{1-\alpha}$  and  $\alpha = \frac{\beta}{\beta+1}$  are the current gains in CE and CB configurations, respectively.

Comparing (10) – (12) it is obvious that in forward active region:

$$i_C = \beta i_B \quad (13)$$

and:

$$i_C = \alpha i_E \quad (14)$$

Actual collector current is somewhat  $v_{CE}$  voltage-dependent what is not modelled in (10) and to account it modified version of (10) is used:

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \left( 1 + \frac{v_{CE}}{V_A} \right) \quad (15)$$

where:  $V_A$  is parameter called Early voltage.

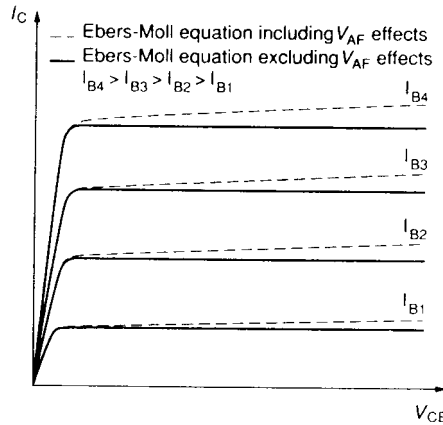


Fig. 7. Typical output characteristics of npn BJT without (solid line) and with Early effect (dashed line).

## 6. Simplified BJT model for operating point calculation

Simplified equations for npn and pnp BJT are summarised in below table. Fig. 8 shows resulting schematic to be used for such calculations in forward active and saturated mode.

Tab. 1. Summarised simplified model of BJT for operating point calculation.

Working region	npn	pnp
Forward active	$I_C = \beta I_B$ $V_{BE} = 0.7V$	$I_C = \beta I_B$ $V_{BE} = -0.7V$
Reverse active	$I_E = \beta_R I_B$ , $V_{BC} = 0.7V$	$I_E = \beta_R I_B$ $V_{BC} = -0.7V$
Forward saturated	$V_{CE} = 0.2V$	$V_{CE} = -0.2V$

	$V_{BE} = 0.7V$	$V_{BE} = -0.7V$
Reverse saturated	$V_{EC} = 0.2V$	$V_{EC} = -0.2V$
	$V_{BC} = 0.7V$	$V_{BC} = -0.7V$
Cutoff	$I_B = I_C = I_E = 0$	$I_B = I_C = I_E = 0$

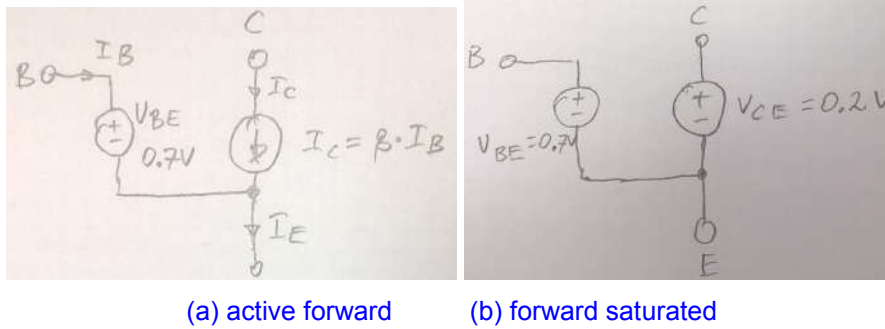


Fig. 8. Equivalent npn model for operating point calculation.

### 7. Hybrid- $\pi$ small-signal BJT model

BJT is most often used as an amplifier working in forward active region. Its small signal model can be derived from (10-12), (15) by finding appropriate derivatives in the operating point:

$$g_m = \left. \frac{\partial I_C}{\partial v_{BE}} \right|_{Q=V_{CE}, V_{BE}, I_C} = I_S e^{\frac{v_{BE}}{V_T}} \left( 1 + \frac{v_{CE}}{V_A} \right) \Big|_{Q=V_{CE}, V_{BE}, I_C} = \frac{I_C}{V_T} \quad (16)$$

$$g_o = \left. \frac{\partial I_C}{\partial v_{CE}} \right|_{Q=V_{CE}, V_{BE}, I_C} = I_S e^{\frac{v_{BE}}{V_T}} \frac{1}{V_{AF}} \Big|_{Q=V_{CE}, V_{BE}, I_C} = \frac{I_C}{V_A} \frac{1}{1 + \frac{v_{CE}}{V_A}} \approx \frac{I_C}{V_A} \quad (17)$$

$$g_\pi = \left. \frac{\partial I_B}{\partial v_{BE}} \right|_{Q=V_{CE}, V_{BE}, I_C} = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} \left( 1 + \frac{v_{CE}}{V_A} \right) \Big|_{Q=V_{CE}, V_{BE}, I_C} = \frac{I_C}{\beta V_T} = \frac{g_m}{\beta} \quad (18)$$

The schematic of the hybrid- $\pi$  model is presented in the Fig. 9.

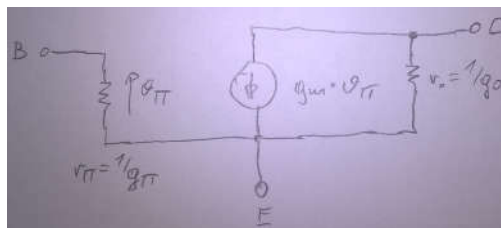


Fig. 9. Small-signal hybrid- $\pi$  BJT model for low frequency. Please note that for both, npn and pnp devices model is identical.

Model presented in Fig. 9 is valid only for low frequency. For higher frequencies it have to be equipped with two junction (BE and BC) capacitances. Such filled model is presented in Fig. 10.

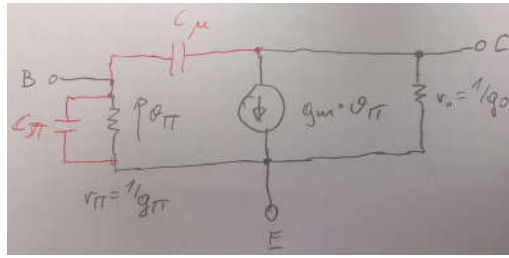


Fig. 10. Small-signal hybrid- $\pi$  BJT model filled with junction capacitances.

Value of base – emitter capacitance can be calculated by:

$$C_{\pi} = \frac{g_m}{2\pi f_T} - C_{\mu} \quad (19)$$

where:  $C_{\mu}$  is the base – collector junction capacitance and  $f_T$  is cutoff frequency of transistor.

## 8. Amplifier analysis procedure

Standard procedure of amplifier analysis is performed in the following steps:

- operating point calculation (DC analysis), schematic for this step is modified by elimination of inductors (by short circuit) and capacitors (by breaks),
- based on OP values calculation of parameters of small-signal models,
- creation of schematic for small-signal analysis, nonlinear devices are replaced by its small-signal models, capacitances and inductors by appropriate impedances (for c – coupled amplifiers capacitors can be roughly treated as short circuits), power supply is treated as signal ground,
- analysis of created model by finding gain (transmittance), input and output resistance (impedance), frequency response and other needed parameters, (AC analysis).

## 9. Examination of CE, CC and CB configuration of BJT amplifier

**Exercise 1.** Please calculate operating point, gain, input and output resistance for the circuit of CE configuration presented below. Please assume that capacitors are short circuits for small-signal component.

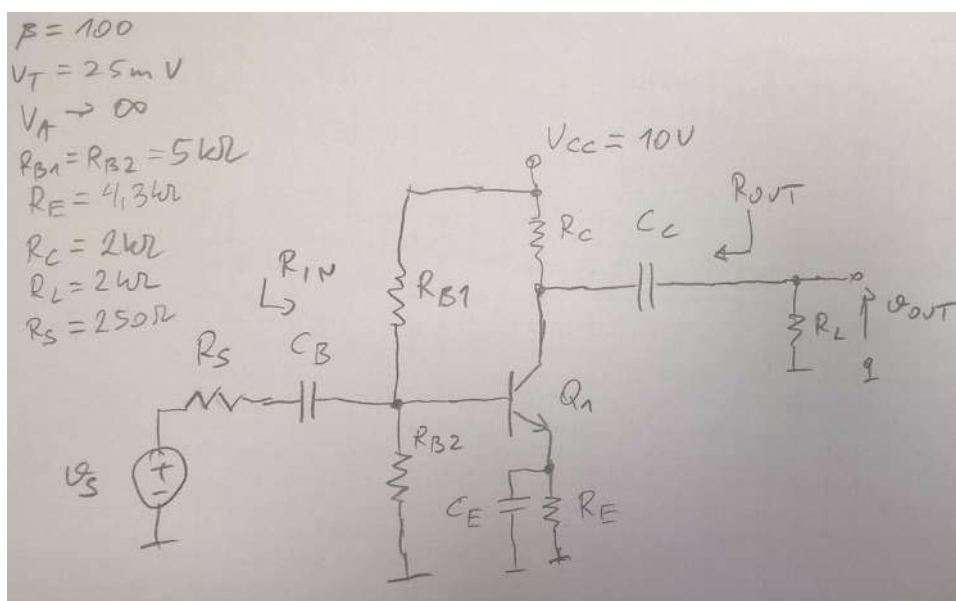


Fig. 11. CE configuration of BJT amplifier.

**Solution:**

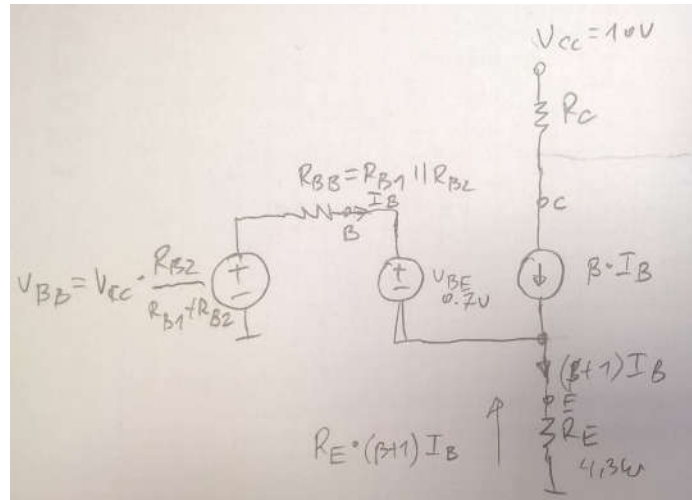


Fig. 12. Schematic for operating point calculation.

From the Fig. 12 base current can be found from the equation of eyelet  $V_{BB}$  – ground. This equation can be written as:

$$V_{BB} = R_{BB} I_B + V_{BE} + (\beta + 1) I_B R_E \quad (20)$$

finding base current and then collector current it gives following values:

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB} + (\beta + 1) R_E} \quad \text{and} \quad I_C = \beta \frac{V_{BB} - V_{BE}}{R_{BB} + (\beta + 1) R_E} = \frac{\beta}{\beta + 1} \frac{V_{BB} - V_{BE}}{R_{BB}/(\beta + 1) + R_E} \quad (21, 22)$$

For our amplifier, assuming:

$\frac{\beta}{\beta + 1} \approx 1$  and  $R_{BB}/(\beta + 1) \ll R_E$  equation (22) simplifies to the form:

$$I_C = \frac{\beta}{\beta + 1} \frac{V_{BB} - V_{BE}}{R_{BB}/(\beta + 1) + R_E} \approx \frac{V_{BB} - V_{BE}}{R_E} \quad (23)$$

Lets calculate values for our amplifier:

$V_{BB} = 5V$ ,  $R_{BB} = 2.5k\Omega$ ,  $I_C = 1mA$  and check if assumption of active forward region is met:

$$V_C = V_{CC} - I_C R_C = 10V - 2k\Omega * 1mA = 8V$$

$$V_E = I_E R_E \approx 4.3k\Omega * 1mA = 4.3V$$

so  $V_{CE} = V_C - V_E = 8V - 4.3V = 3.7V > 0.2V$  and Q1 is in active forward region.

Parameters of small-signal hybrid- $\pi$  model:

$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = 40mS \quad (24)$$

$$r_\pi = \frac{\beta_c}{g_m} = \frac{100}{40mS} = 2.5k\Omega \quad (25)$$



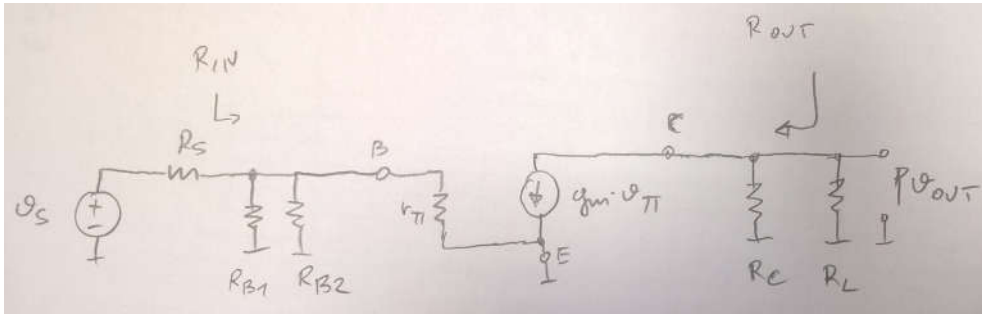


Fig. 13. Small-signal equivalent model for the amplifier from Fig. 11.

Parameters derived from small-signal model in Fig. 13 are as follows:

Voltage gain:

$$\frac{v_{out}}{v_s} = -\frac{R_{B1} \parallel R_{B2} \parallel r_{\pi}}{R_s + R_{B1} \parallel R_{B2} \parallel r_{\pi}} \cdot gm(R_C \parallel R_L) = -\frac{5k \parallel 5k \parallel 2.5k}{250 + 5k \parallel 5k \parallel 2.5k} \cdot 40mS \cdot (2k \parallel 2k)\Omega = -\frac{1250}{1500} \cdot 40 = -33.3V/V \quad (26)$$

Input resistance:

$$R_{IN} = R_{B1} \parallel R_{B2} \parallel r_{\pi} = 5k\Omega \parallel 5k\Omega \parallel 2.5k\Omega = 1250\Omega \quad (27)$$

Output resistance:

$$R_{OUT} = R_C = 2k\Omega \quad (28)$$

**Exercise 2.** Please calculate operating point, gain, input and output resistance for the circuit of CC configuration presented below. For calculations please use component values identical as in previous exercise. Please assume that capacitors are short circuits for small-signal component.

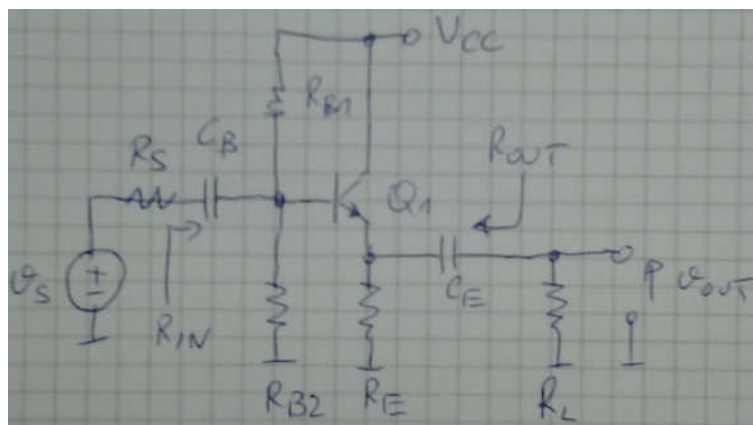


Fig. 14. CC configuration of BJT amplifier.

**Solution:**

Operating points of circuits from Fig. 14 and 11 are identical so calculations will not be repeated. Small-signal equivalent model for amplifier from Fig. 14 is presented in the Fig. 15. Parameters derived from this model are as follows:

Lets calculate  $R_{IN}$  as a first to the use it in the input signal voltage divider:

$$R_{IN} = R_{B1} \parallel R_{B2} \parallel [r_{\pi} + (\beta + 1)(R_E \parallel R_L)] = 5k\Omega \parallel 5k\Omega \parallel (2.5k\Omega + 101(4.3k\Omega \parallel 2k\Omega)) = 2.5k\Omega \parallel (2.5k\Omega + 137.87k\Omega) = 2.46k\Omega \approx 2.5k\Omega \quad (29)$$

Then voltage gain can be estimated as:

$$\frac{v_{out}}{v_s} = \frac{R_{IN}}{R_s + R_{IN}} \cdot \frac{gm(R_E \parallel R_L)}{1 + gm(R_E \parallel R_L)} = \frac{2.46k\Omega}{250\Omega + 2.46k\Omega} \cdot \frac{40mS \cdot (4.3k\Omega \parallel 2k\Omega)}{1 + 40mS \cdot (4.3k\Omega \parallel 2k\Omega)} = \frac{2.46k\Omega}{2.71k\Omega} \cdot \frac{54.6}{55.6} = 0.891V/V \quad (30)$$

And output resistance is equal to:

$$R_{OUT} = R_E \parallel (\text{resistance\_seen\_from\_Emitter}) = R_E \parallel \left[ \frac{(R_s \parallel R_{B1} \parallel R_{B2}) + r_{\pi}}{\beta + 1} \right] = 4.3k\Omega \parallel \left[ \frac{(250\Omega \parallel 5k\Omega \parallel 5k\Omega) + 2.5k\Omega}{100 + 1} \right] = 4.3k\Omega \parallel \left[ \frac{(250\Omega \parallel 5k\Omega \parallel 5k\Omega) + 2.5k\Omega}{100 + 1} \right] = 4.3k\Omega \parallel 27\Omega = 26.83\Omega \quad (31)$$

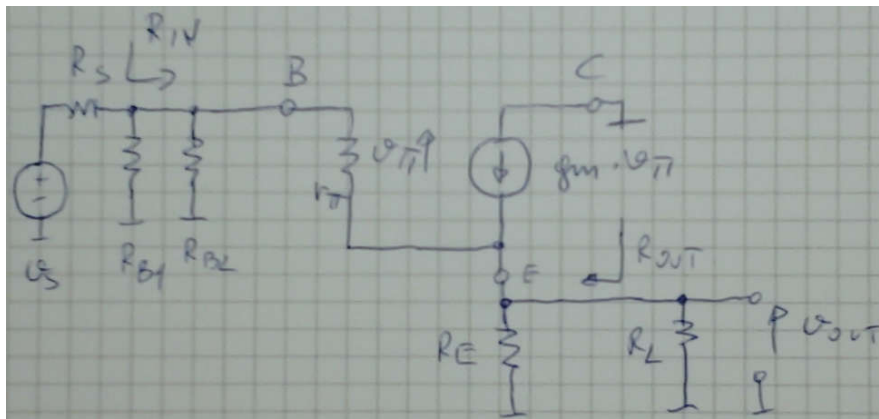


Fig. 15. Small-signal equivalent model for the amplifier from Fig. 14.

**Exercise 3.** Please calculate operating point, gain, input and output resistance for the circuit of CB configuration presented below. For calculations please use component values identical as in previous exercise. Please assume that capacitors are short circuits for small-signal component.

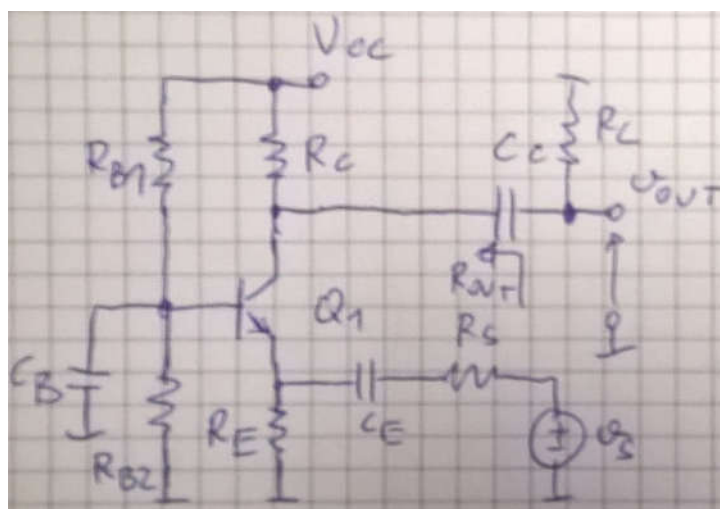


Fig. 16. BJT amplifier in CB configuration.

**Solution:**

Operating points of circuits from Fig. 16 and 11 are identical so calculations will not be repeated. Small-signal equivalent model for amplifier from Fig. 16 is presented in the Fig. 17. Parameters derived from this model are as follows:

Lets calculate  $R_{IN}$  as a first to the use it in the input signal voltage divider:

$$R_{IN} = R_E \parallel \left( \text{resistance\_seen\_from\_Emitter} \right) = R_E \parallel \left( \frac{r_\pi}{\beta + 1} \right) =$$

$$= 4.3k\Omega \parallel \left( \frac{2.5k\Omega}{100 + 1} \right) = 4.3k\Omega \parallel 24.75\Omega = 24.61\Omega \quad (32)$$

Then voltage gain can be estimated as:

$$\frac{v_{out}}{v_s} = \frac{R_{IN}}{R_s + R_{IN}} \cdot gm(R_C \parallel R_L) = \frac{24.61\Omega}{250\Omega + 24.61\Omega} \cdot 40mS \cdot (2k\Omega \parallel 2k\Omega) =$$

$$= 0.0896 \cdot 40 = 3.584V/V \quad (33)$$

And output resistance is equal to:

$$R_{out} = R_C = 2k\Omega \quad (34)$$

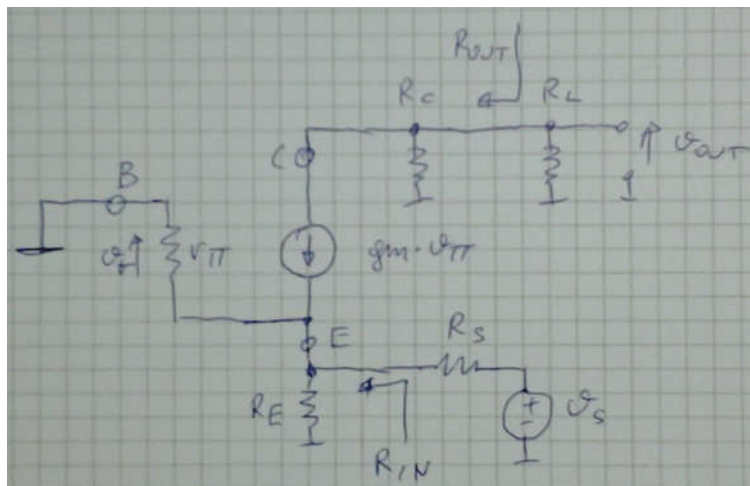


Fig. 17. Small-signal equivalent model for the amplifier from Fig. 16.

## 10. Current mirror subcircuit

Current mirror is a subcircuit which generates current approximately identical such as given to its input. Below presented is the simplest form of such a circuit. There is also possibility to multiply output current if not identical devices are used.

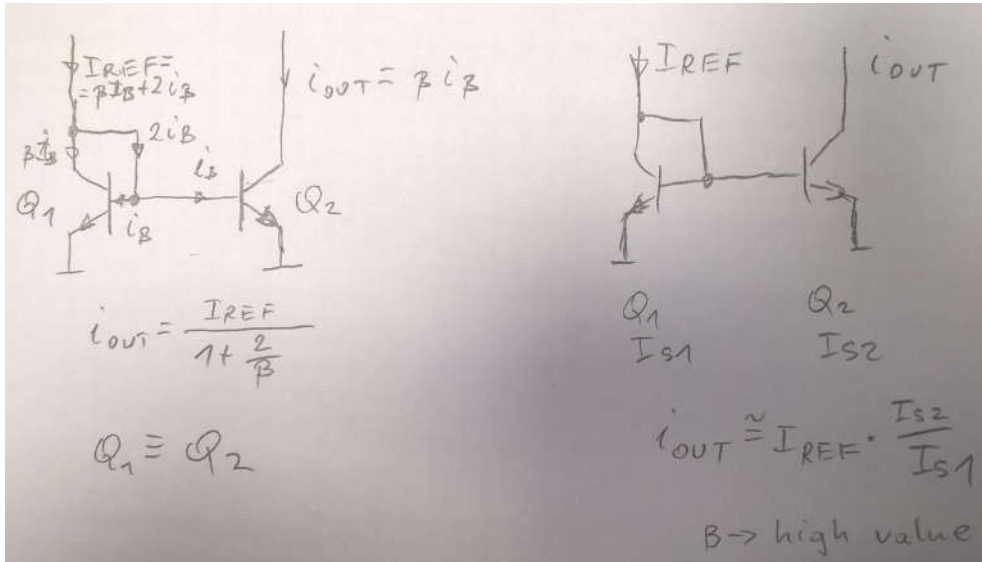


Fig. Bipolar current mirror built of identical and different transistors.

For the circuit above, small-signal output resistance is equal to resistance of Q2 device and equal to:

$$r_{out} = \frac{V_A}{I_c} = \frac{V_A}{i_{out}} \quad (35)$$

## 11. Bipolar differential pair

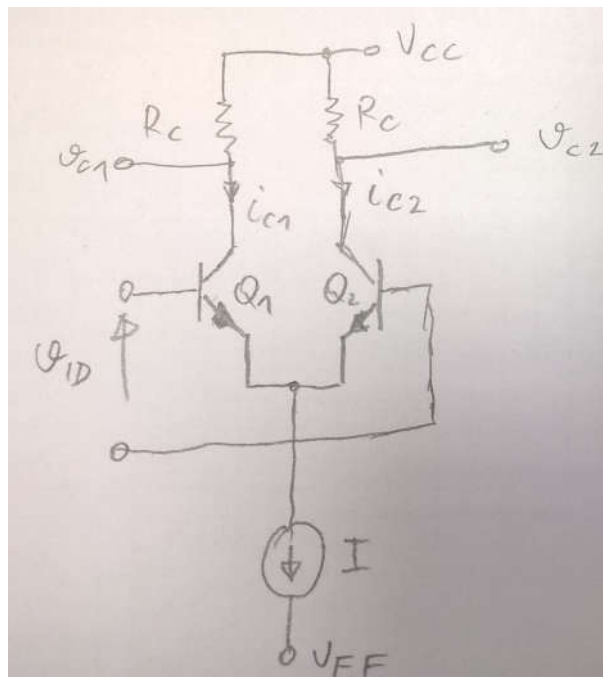


Fig. 18. Bipolar differential pair loaded by resistor  $R_c$ .

DC analysis of differential pair from Fig. 14 gives following results:

$$I_{c1} = I_{c2} = I / 2 \quad (36)$$

$$V_{c1} = V_{c2} = V_{cc} - \frac{1}{2} I R_c \quad (37)$$

Thus small-signal Q1 and Q2 parameters are identical and equal to:

$$g_m = \frac{1}{2} \frac{I}{V_T} \quad r_\pi = \frac{2\beta V_T}{I} \quad \text{and} \quad r_o = \frac{2V_A}{I} \quad (38, 39, 40)$$

AC analysis gives following results:

Gains to collectors of transistors:

$$A_{d,c1} = \frac{v_{c1}}{v_{id}} = -\frac{1}{2} g_m (R_c \parallel r_o) = -\frac{I}{4V_T} \left( R_c \parallel \frac{2V_A}{I} \right) \quad (41)$$

$$A_{d,c2} = \frac{v_{c2}}{v_{id}} = \frac{1}{2} g_m (R_c \parallel r_o) = \frac{I}{4V_T} \left( R_c \parallel \frac{2V_A}{I} \right) \quad (42)$$

Input differential resistance:

$$R_{id} = 2r_\pi = \frac{4\beta V_T}{I} \quad (43)$$

Output resistances to collectors of transistors:

$$R_{out} = R_c \quad (44)$$

For differential pair also common – mode gain is under consideration. Common-mode gain is calculated for both inputs tied together to common input signal. For circuit from Fig. 14, because of ideal current source  $I$ , this gain is equal to 0. Nevertheless, if real current source with finite resistance of value  $r_{CS}$  is used then CM gain to both collector is the same and can be found to be equal to:

$$A_{CM} = \frac{v_{c1}}{v_{CM}} = \frac{v_{c2}}{v_{CM}} = -\frac{g_m}{1 + 2g_m r_{CS}} (R_c \parallel r_o) \quad (45)$$

Common – mode gain is undesirable and to characterize differential amplifier Common Mode rejection Factor (CMRR) was introduced as absolute ratio of differential to common mode gain:

$$CMRR = \left| \frac{A_d}{A_{CM}} \right| = \frac{1 + 2g_m r_{CS}}{2} \approx g_m r_{CS} \quad (46)$$

If current source  $I$  is built as simple current mirror then CMRR can be estimated as:

$$CMRR \approx g_m r_{CS} = \frac{I}{2V_T} \frac{V_A}{I} = \frac{V_A}{2V_T} \quad (47)$$

## 12. Analysis of simple bipolar operational amplifier (OA)

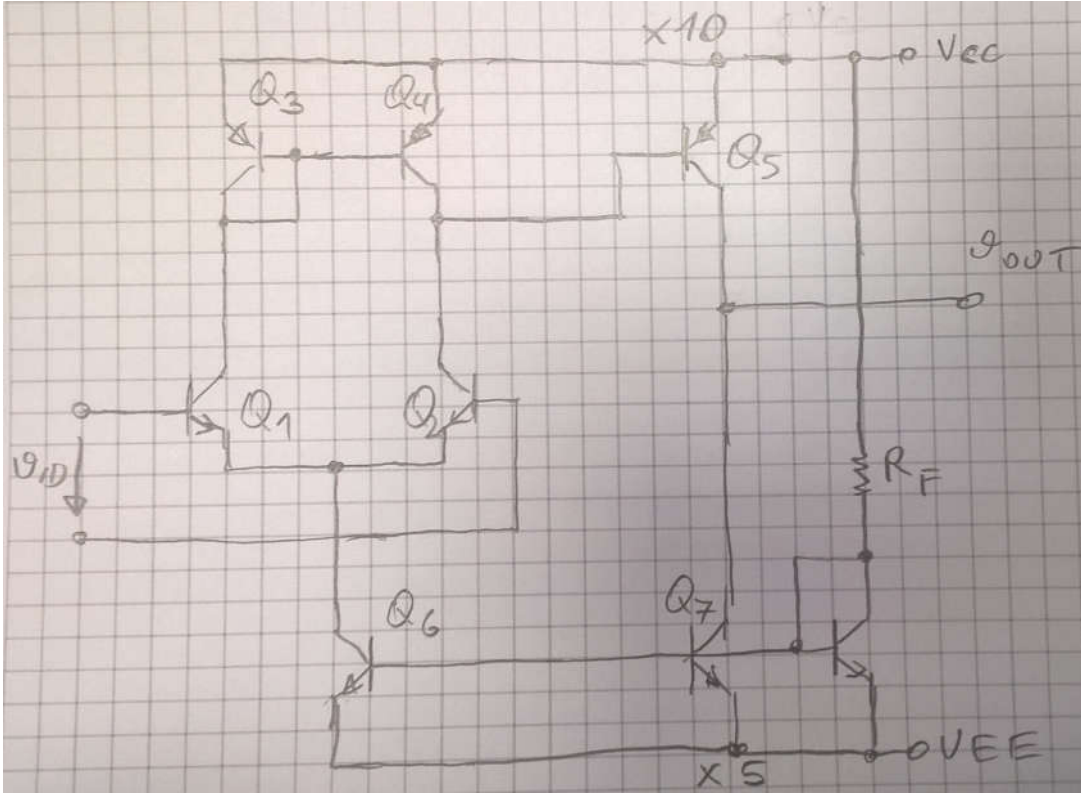


Fig. 19. Simple bipolar operational amplifier.

DC analysis of amplifier from Fig. 15 gives following results:

Current through  $R_F$ :

$$I_{RF} = \frac{V_{CC} - V_{EE} - V_{BE}}{R_F} \quad (48)$$

So collector currents are equal to (neglecting base currents):

$$I_{RF} = I_{C8} = I_{C6}, \quad I_{C7} = I_{C5} = 5I_{RF}, \quad I_{C1} = I_{C2} = I_{C3} = I_{C4} = \frac{1}{2}I_{RF} \quad (49, 50, 51)$$

Thus small-signal Q1 and Q2 parameters are identical and equal to:

$$g_{m12} = \frac{1}{2} \frac{I_{RF}}{V_T}, \quad r_{\pi12} = \frac{2\beta V_T}{I_{RF}} \quad (52, 53)$$

And small-signal parameter of second stage built on Q5 is following:

$$g_{m5} = \frac{5I_{RF}}{V_T}, \quad r_{\pi5} = \frac{\beta V_T}{5I_{RF}} \quad \text{and} \quad r_{o5} = \frac{V_A}{5I_{RF}} \quad (54, 55, 56)$$

Additionally output resistance of Q7:

$$r_{o7} = \frac{V_A}{5I_{RF}} \quad (57)$$

AC analysis gives following results:

Differential gain:

$$A_D = \frac{v_{\pi 5}}{v_{ID}} \frac{v_{OUT}}{v_{\pi 5}} = (-g_{m12} r_{\pi 5}) [-g_{m5} (r_{o5} \parallel r_{o7})] = \left( -\frac{I_{RF}}{2V_T} \frac{\beta V_T}{5I_{RF}} \right) \left( -\frac{5I_{RF}}{V_T} \frac{V_A}{10I_{RF}} \right) =$$

$$= \left( -\frac{\beta}{10} \right) \left( -\frac{V_A}{2V_T} \right) \quad (58)$$

For typical values of  $\beta=100$ ,  $V_T=25mV$  and  $V_A=100V$  equation (47) gives result:

$$A_D = \left( -\frac{\beta}{10} \right) \left( -\frac{V_A}{2V_T} \right) = \left( -\frac{100}{10} \right) \left( \frac{100V}{50mV} \right) = 20000V/V \quad (59)$$

Please note that gain is not dependent on  $I_{RF}$ .

Input differential resistance as for differential pair is equal to:

$$R_{ID} = 2r_{\pi 12} = \frac{4\beta V_T}{I_{RF}} \quad (60)$$

For  $I_{RF}=1mA$  this resistance is equal to:

$$R_{ID} = \frac{4\beta V_T}{I_{RF}} = \frac{4 \cdot 100 \cdot 25mV}{1mA} = 10k\Omega \quad (61)$$

Output resistance can be estimated as Q5 and Q7 in parallel:

$$R_{OUT} = r_{o7} \parallel r_{o7} = \frac{V_A}{10I_{RF}} \quad (62)$$

For  $I_{RF}=1mA$  this resistance is equal to:

$$R_{OUT} = \frac{V_A}{10I_{RF}} = \frac{100V}{10 \cdot 1mA} = 10k\Omega \quad (63)$$

Because simple current mirror is used for biasing differential pair then CMRR can be estimated as:

$$CMRR \approx \frac{V_A}{2V_T} = \frac{100V}{2 \cdot 25mV} = 2000 \quad (64)$$

### 13. Parameters of ideal and real operational amplifier

Ideal operational amplifier is differential voltage controlled voltage source. Thus its ideal parameters are as of such type of source. Symbol of operational amplifier together with its main properties is presented in the Fig. 20. Below table summarises parameters of ideal OA as well as typical parameters of commercially available OA.

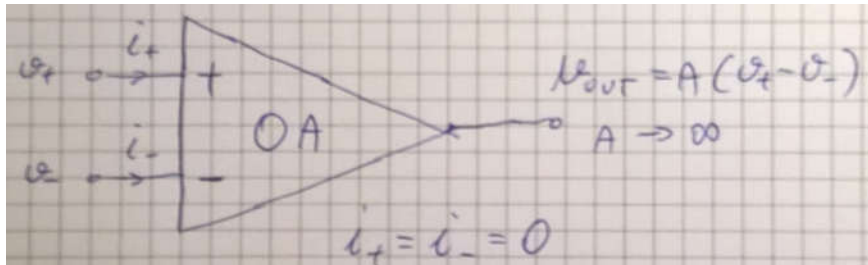


Fig. 20. Symbol of operational amplifier (OA).

Tab. 2. Summarised main parameters of ideal and typical OA.

Parameter name	Unit	Ideal OA	Typical range
Differential gain $A_D$	V/V	infinity	$10^4 - 10^7$
CMRR	dB	infinity	60 - 120
Input resistance	M $\Omega$	infinity	0.1 - 10 for BJT,

			tens of $G\Omega$ for CMOS
Output resistance	$\Omega$	0	10 - 1000
3dB bandwidth	Hz	infinity	10 - 10.000
Slew Rate	V/ $\mu$ s	infinity	.1 - 10

## 14. Application of operational amplifier

Because of very high differential gain OA working in negative feedback loop exhibits so called virtual short circuit between both inputs. It can be easily explained by transformation OA equation to the form:

$$(v_+ - v_-) = v_{ID} = \frac{v_{OUT}}{A} \Big|_{A \rightarrow \infty} = 0 \quad (65)$$

**Non-inverting amplifier:**

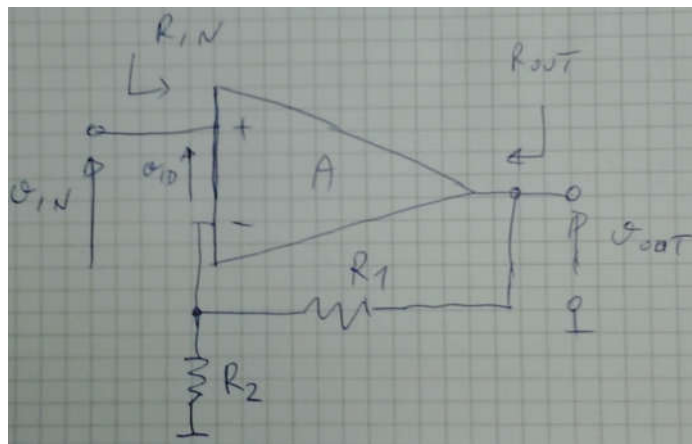


Fig. 21. Non-inverting amplifier with the use of OA.

Voltage gain:

$$\frac{v_{OUT}}{v_{IN}} = 1 + \frac{R_1}{R_2} \quad (66)$$

Input resistance:

$$R_{IN} = \infty \quad (67)$$

Output resistance

$$R_{OUT} = 0 \quad (68)$$

**Inverting amplifier:**



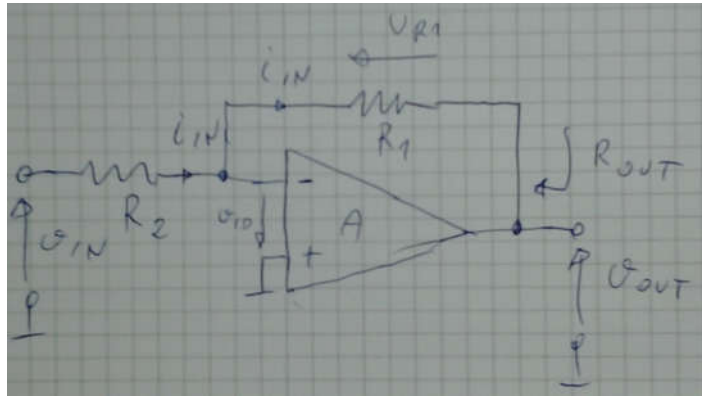


Fig. 22. Inverting amplifier with the use of OA.

Voltage gain:

$$\frac{v_{OUT}}{v_{IN}} = -\frac{R_1}{R_2} \quad (69)$$

Input resistance:

$$R_{IN} = R_2 \quad (70)$$

Output resistance

$$R_{OUT} = 0 \quad (71)$$

**Voltage summer:**

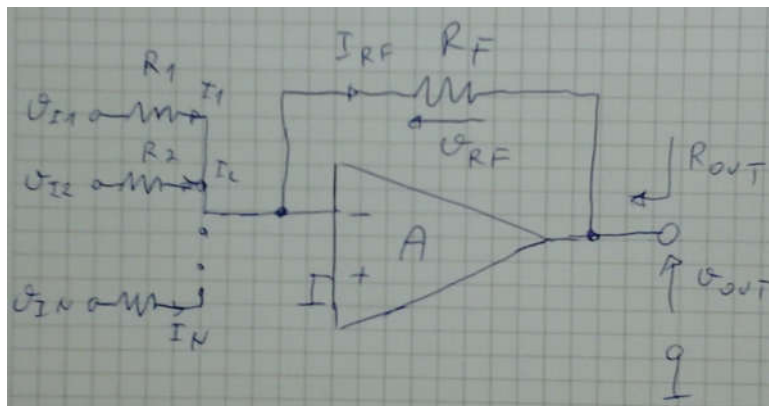


Fig. 23. Voltage summer.

Output voltage:

$$v_{OUT} = -R_F \left( \frac{v_{I1}}{R_1} + \frac{v_{I2}}{R_2} + \dots + \frac{v_{IN}}{R_N} \right) = -R_F \sum_{i=1}^N \frac{v_{Ii}}{R_i} \quad (72)$$

Input resistance:

$$R_{IN} = R_2 \quad (73)$$

Output resistance

$$R_{OUT} = 0 \quad (74)$$