Continuous-Time Integrated Filters

First semester of second level studies

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Assessment methods and criteria for the subject of ”C-TIF”

- Lecture - written exam.
- Laboratory - 5 simulation exercises (PSPICE).
- Final grade - weighted average of the exam grade (weight 3) and laboratory (weight 1).
- For those who are interested, there will be a extra exam with 15% attendance at lectures, it will be during last hour of lecture or immediately after last lecture.

Literature:

[1] R. Schaumann, M. E. Van Valkenburg, „ Design of analog filters", Oxford University Press 2001.

[2] K. R. Laker, W. M. C. Sansen, „Design of analog integrated circuits and systems", McGraw-Hill, Inc. 1994.

[3] G.Feri, N.C. Guerrini, „Low-voltage low-power novel CCII topologies and applications", In Proc. of ICECS 2001, pp. 1095-1098 vol. 2, 2001.

[4] M. Białko, A. Guziński, W. Sieńko, J. Żurada, „Filtry aktywne RC" WNT, Warszawa 1979.

[5] M.E. Van Valkenburg, „Analog Filter Design",CBS College Publishing 1982

List of topics - lecture

- Introduction, classification of continuous-time active filters.
- Building blocks and properties of operational amplifiers (i.e. Amps, OTAs and operational transresistance amplifiers).
- Current transformation, the second generation current convejors (CCII).
- Introduction to synthesis of active filters, normalization procedures, frequency transformations, approximation methods.
- The synthesis of second-order active filters.

List of topics – lecture, cont.

- Cascade realizations of high-order filters.
- Circuit methods for grounded and floating inductor realizations.
- Methods for LC ladder simulations.
- The design of current-mode filters.
- The design of integrated continuous-time fully-differential high-order OTA-C and Gm-C filters.

List of topics – lecture, cont.

- Multiple-loop feedback structure realizations.
- Realization of LC ladder using gyrator structures.
- Realization of LC ladder using signal flow graph synthesis.
- Sensitivity, noise, nonlinear distortion and dynamic range considerations.
- Automatic tuning circuitry and programming.

List of topics – laboratory

- PSPICE simulation of the CMOS Operational Transconductance Amplifier (OTA).
- PSPICE simulation of the second generation Current Conveyor (CCII).
- PSPICE simulation of the six order Cascade Filter.
- PSPICE simulation of the six order Gm-C Filter .
- Comparison investigation of properties of the Cascade Filter and based on LC prototype realization.

Types of filters based on different criteria

- passive and active,
- analogue and digital,
- continuous–time and discrete time,
- integrated and discrete,
- lowpass, highpass, bandpass, bandstop and allpass,
- other criteria...

Basic definitions [1]

v_1 – voltage source, v_2 – output voltage, for harmonic signals in a steady state:

$$v_1(t) = V_1 \cos(\omega t + \theta_1) \qquad v_2(t) = V_2 \cos(\omega t + \theta_2)$$

alternatively in complex vector notation written as (the upper index in the form of a dash means complex value):

$$\overline{V_1} = \left|\overline{V_1}\right| e^{j\theta_1} \qquad \overline{V_2} = \left|\overline{V_2}\right| e^{j\theta_2}$$

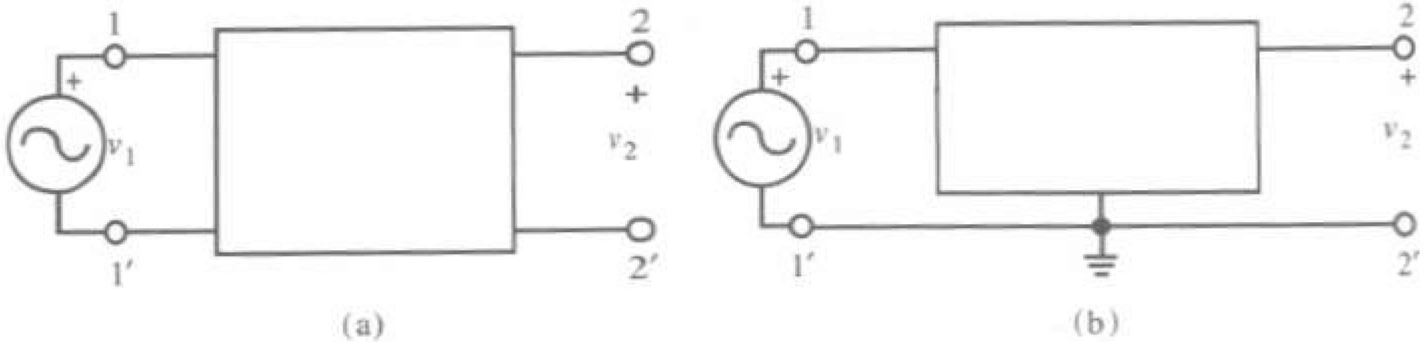


Fig. 1. Two-port network a) with independent input and output nodes and b) with common node of signal ground [1].

Basic definitions-cont. [1]

Input and output voltages can be also presented using Laplace transformations of complex variable s, where $s=\sigma+j\omega$, for harmonic signals in steady state substitution $s=j\omega$ can be used what in turn lead to:

$$\overline{V_1} = \overline{V_1}(s)\Big|_{s=j\omega} = \left|\overline{V_1}(j\omega)\right| e^{j\theta_1(j\omega)}$$

$$\overline{V_2} = \overline{V_2}(s)\Big|_{s=j\omega} = \left|\overline{V_2}(j\omega)\right| e^{j\theta_2(j\omega)}$$

Transmittance of the circuit is equal to:

$$T(s) = \frac{\overline{V_2}(s)}{\overline{V_1}(s)}$$

what in steady state turns to =>

Basic definitions-cont. [1]

$$T(s)\Big|_{s=j\omega} = \frac{\overline{V_2}(s)}{\overline{V_1}(s)}\Bigg|_{s=j\omega} = \frac{\left|\overline{V_2}(j\omega)\right| e^{j\theta_2(\omega)}}{\left|\overline{V_1}(j\omega)\right| e^{j\theta_1(\omega)}} = \frac{\left|\overline{V_2}(j\omega)\right|}{\left|\overline{V_1}(j\omega)\right|} e^{j[\theta_2(\omega)-\theta_1(\omega)]}$$

Amplitude response:

$$\left|T(j\omega)\right| = \frac{\left|\overline{V_2}(j\omega)\right|}{\left|\overline{V_1}(j\omega)\right|} \qquad => \qquad \left|\overline{V_2}(j\omega)\right| = \left|\overline{V_1}(j\omega)\right| \left|T(j\omega)\right|$$

Phase response:

$$\theta(\omega) = \theta_2(\omega) - \theta_1(\omega) \qquad => \qquad \theta_2(\omega) = \theta_1(\omega) + \theta(\omega)$$

Basic definitions-cont. [1]

In the further part of the materials the marking of the complex symbols using upper dash has been omitted.

In the literature, the logarithmic measure of transmittance is often used:

$$\alpha(\omega) = 20\log\left|T(j\omega)\right| \quad [\mathrm{dB}]$$

For α bigger than 0, the system amplifies the signal, while for α less than 0, the system introduces attenuation. Although for lossy circuits, α is less than zero, its absolute value is often given by calling α as loss factor inted of gain factor. In the further part of the materials, the sign next to α should be taken from the context.

Basic definitions-cont. [1]

Table 1. Values of loss and gain according to transmittance absolute value [1].

Loss		Gain	
α [dB]	$\lvert T(j\omega)\rvert$	α [dB]	$\lvert T(j\omega)\rvert$
−100	10^{-5}	100	10^{5}
−60	10^{-3}	60	10^{3}
−20	0.1	20	10
−6	0.501	6	1.995
−3	0.707	3	1.414
−1	0.891	1	1.122
−0.1	0.989	0.1	1.011

Filters types according to amplitude response [1]

Filters for certain frequencies have so called pass band and stop band. Ideally pass band is the frequency range for which $|T|=1$ or similarly $\alpha=0$, and stop band is the frequency range for which $|T|=0$ and similarly $\alpha=-\infty$.

Besides typical amplitude responses shown on next slides, other responses are used in real life. Such responses can be realised as combinations of typical responses.

Actually, ideal responses are not realisable but instead filter real amplitude response is continuous function presented at the slides as dashed line lying next to ideal solid responses.

Filters types according to amplitude response –cont.[1]

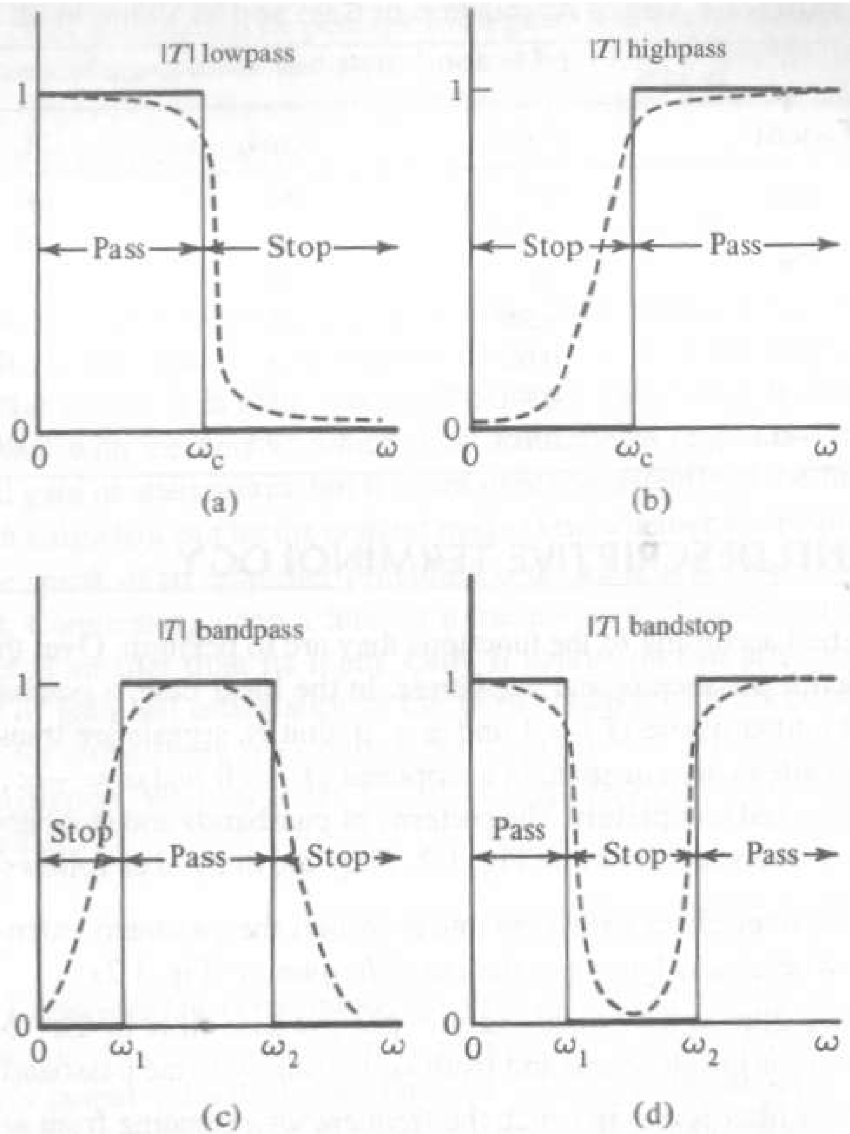


Fig. 2. Basic types of amplitude filter responses, ideal response (solid line) and real response (dahsed line):

a) lowpass,

b) hihgpass,

c) bandpass,

d) bandstop [1].

Filter types-cont. [1]

Actually, filters have transmittances described as ratio of polnynomial of nominator N(s) and polynomial of denominator D(s):

$$T(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

where: a_i, $i = 1,\ldots n$ and b_j, $j = 1,\ldots m$ are real numbers. Coefficient a_n can be set as equal to 1 through nominator and denominator division by a_n. Coefficients of the nominator can be positive, negative or equal to 0. All coefficients of denominator a_i have to be positive – otherwise circuit can be unstable. Order of denominator have to be equal or grater than order of nominator.

Because transmittance and its derivatives are continuous functions it is impossible to realize ideal characteristics show in Fig. 2.

Frequency responses of real filters

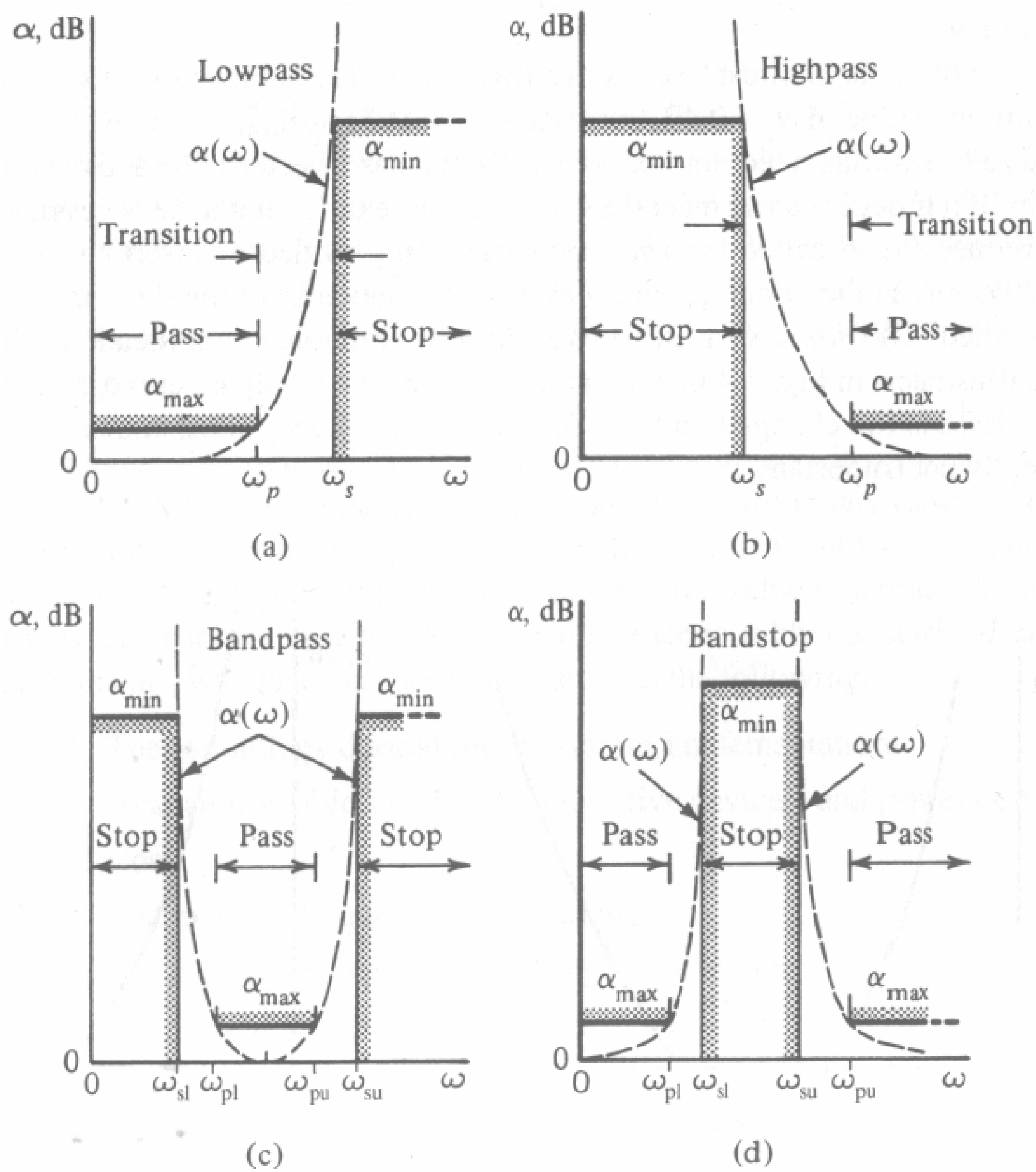


Fig. 3. **Filter specifications** and actually obtained filter responses [1]. Vertical axis is the loss of the filter in logarithmic scale while horizontal axis is the radial frequency.

Approximation is the process of finding of transmittance which satisfies filter specifications.

Synthesis is the process of finding of physical realisation of required transmittance.

Inserted loss/gain of the filter while maintaining general response shape

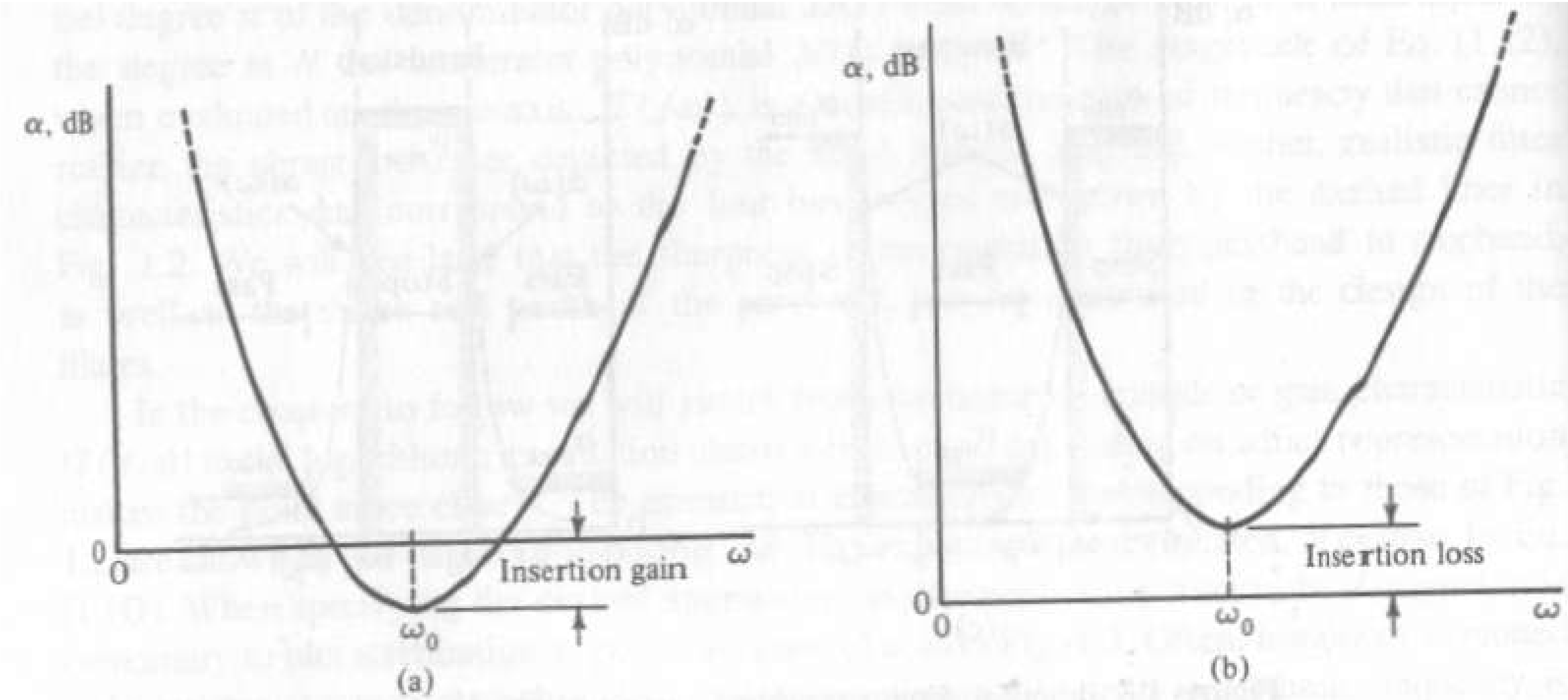


Fig.4. Passband filter with extra gain (a) and loss (b) [1].

Why to use analogue filters in the era of digital signal processing?

Today applications of analogue filters:

• as antialiasing / smoothing filters, in this area analogue filters are not replaceable!

• for very high frequencies,

• in applications justified by costs.

Factors determining the type of used filter [1]

- the technology required,
- power supply voltages and power consumption,
- the cost of implementation,
- frequency ranges of operation,
- stability and sensitivity to parameter changes,
- the weight and dimensions of the filter,
- noise and dynamic range.

Analogue filters frequency of operation

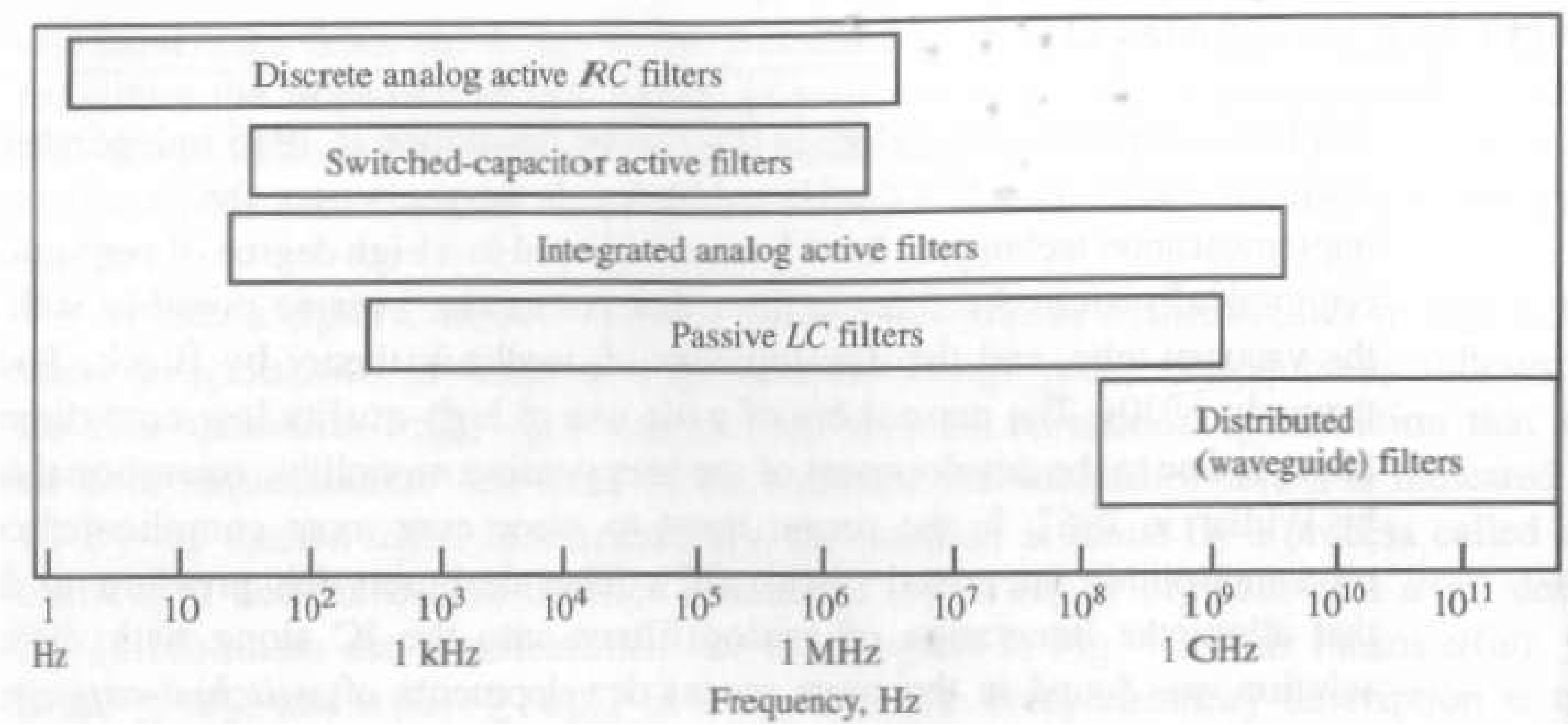


Fig. 5. Types and frequency ranges of analogue filters [1].

Passive devices in integrated circuits [1]

	Discrete	Integrated
Tolerances	1–20%	10–40% absolute 0.1–1% for ratios
Resistors		
Preferred range	1–100 kΩ	Process dependent: values
Lower limit	0.05–1 kΩ	with 10% to 30% tolerances
Upper limit	100–500 kΩ	in the range of 50 Ω–1 kΩ
Capacitors		
Readily realizable	5 pF–1 μF	0.5–5 pF
Practical	0.5 pF–10 μF	0.2–10 pF
Marginally practical	0.2 pF–500 μF	0.1–50 pF
Inductors		
Readily realizable	1 μH–10 mH	Real inductors with large losses
Practical	0.1 μH–50 mH	of the order of 10 nH or less
Marginally practical	100 nH–1 H	

Normalisation of devices values and frequency

Normalised values are calculated according to the following equations (they are dimensionless):

$$L_n = \frac{\omega_S}{R_S} L \qquad C_n = \omega_S R_S C \qquad R_n = \frac{1}{R_S} R$$

where: R_S – normalising resistance, ω_s – normalising angular frequency.

Having normalised values real ones can be calculated by:

$$L = \frac{R_S}{\omega_S} L_n \qquad C = \frac{1}{R_S \omega_S} C_n \qquad R = R_S R_n$$

Normalisation of devices values and frequency – cont.

Normalisation / denormalisation allows the analysis of filters around a normalized pulsation chosen as equal to 1rad / sec.

Normalisation does not change the shape of transmittance but only transfers to other (usually lower) frequencies.

Using the normalisation and denormalisation process for various values of normalising resistance, we have an additional field of freedom in the selection of the actual values of elements R, L, C.

Amplifiers used in integrated circuits

- voltage operational amplifier (OA),
- transconductance amplifier (OTA),
- current conveyor, second generation current conveyor (CCII),

Parameters of ideal and typical OA [1]

Ideally OA is a voltage controlled voltage source having following parameters:

- infinitive differential voltage gain,

- common mode gain equal to zero,

- infinitive input resistance,

- output resistance equal to zero,

- infinitive values of: bandwidth, output current and voltage limit, slew rate, input and output signal ranges...

OA–cont. [1]

Typical internal schematic of popular ”741” OA is presented below [1].

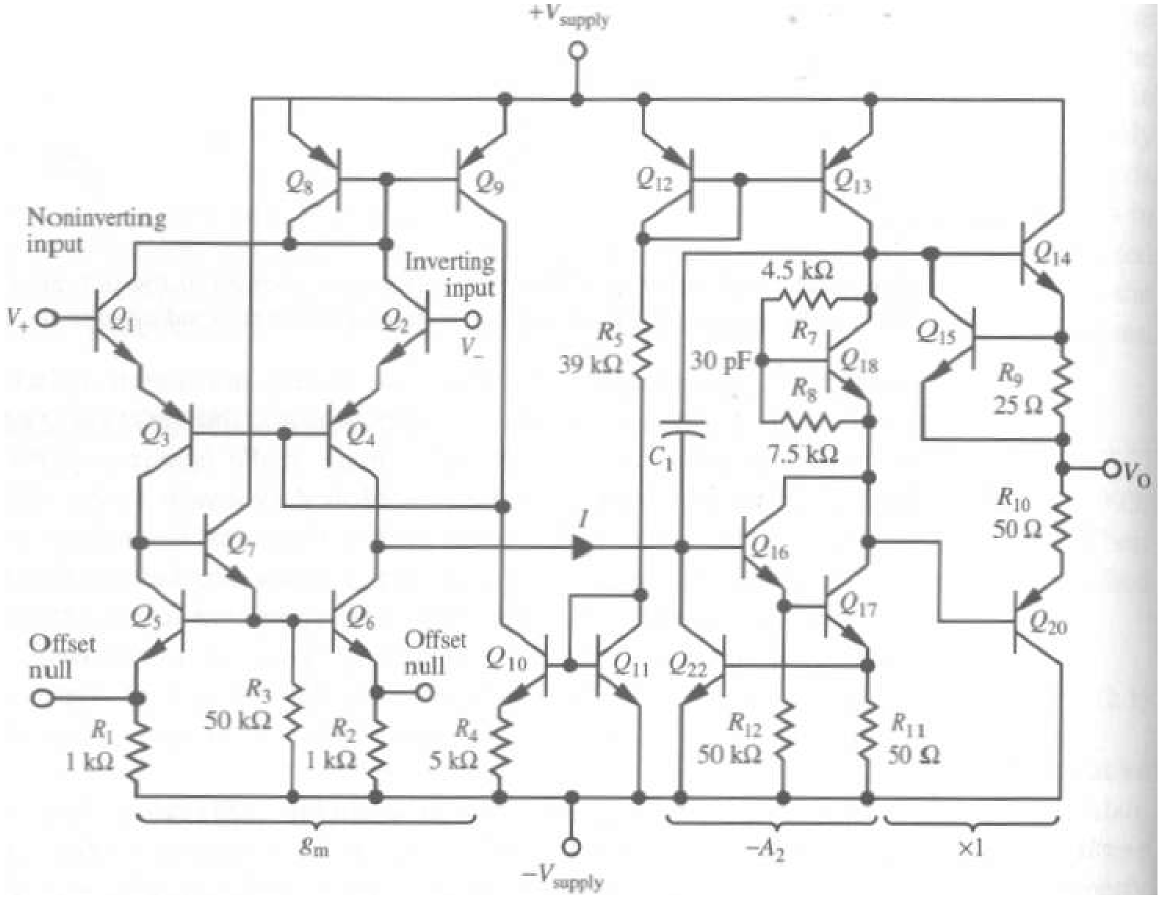


Fig. 6. Bipolar OA [1] of type ”741”.

OA–cont. [1]

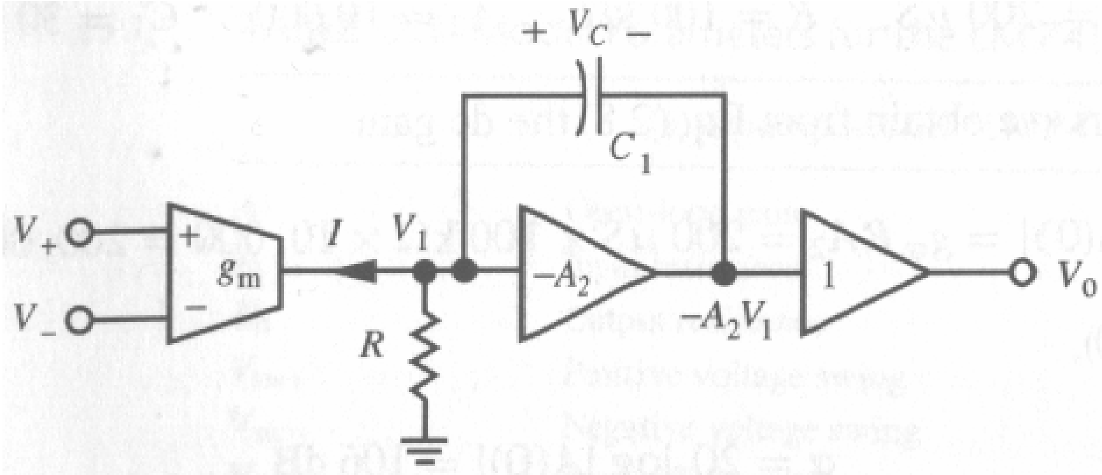


Fig. 7. Simplified model of OA from fig.6.

$$A(s) = \frac{V_0(s)}{V_+(s) - V_-(s)} = \frac{g_m}{sC_1(1 + 1/A_2) + 1/(RA_2)}\Bigg|_{1 >> 1/A_2}$$

$$\approx \frac{g_m}{sC_1 + 1/(RA_2)} = \frac{g_m / C_1}{s + 1/(RC_1A_2)}$$

OA–cont. [1]

Direct current gain (and for small frequencies):

$$A(0) = g_m R A_2$$

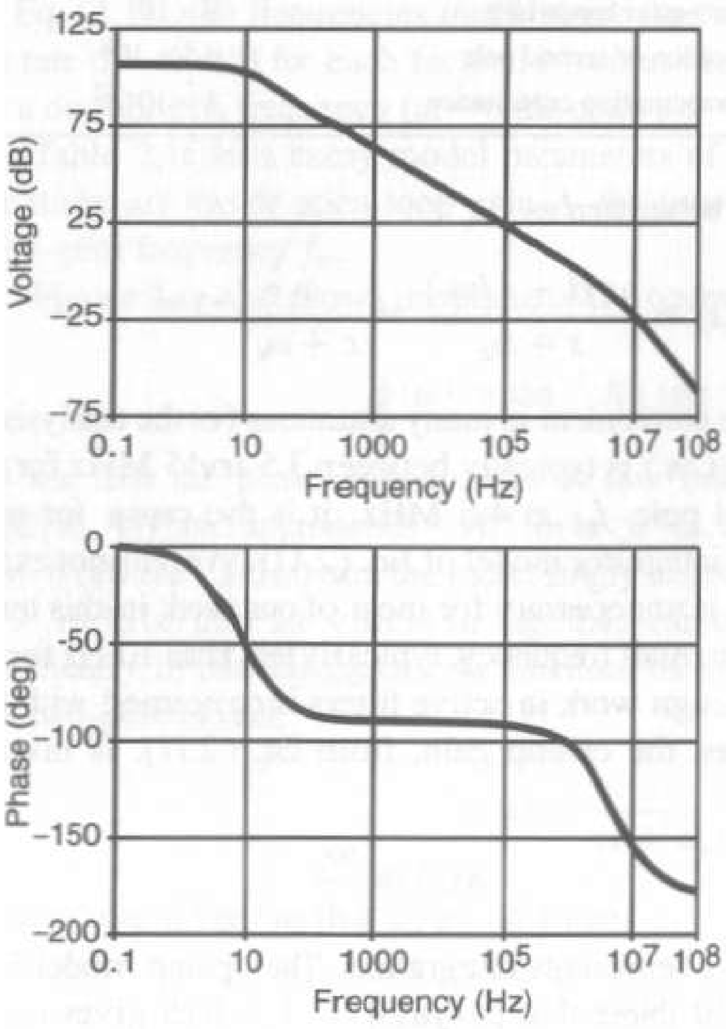
3dB angular frequency:

$$\omega_a = \frac{1}{R C_1 A_2}$$

Cutoff angular frequency:

$$\omega_t = \frac{g_m}{C_1} = A(0)\omega_a$$

OA–cont. [1]



A	Open-loop gain	2×10^5	
R_i	Input resistance	2×10^6	Ω
R_o	Output resistance	75	Ω
V_{sw+}	Positive voltage swing	21	V
V_{sw-}	Negative voltage swing	−21	V
V_{os}	Input offset voltage	0.001	V
I_{bs}	Input bias current	8×10^{-8}	A
I_{os}	Input offset current	2×10^{-8}	A
SR	Slew rate	5×10^5	V/s
f_u	Unity-gain bandwidth	1.5×10^6	Hz
f_{p2}	Location of second pole	4.5×10^6	Hz
C_c	Compensation capacitance	3×10^{-11}	F

Fig. 8. Frequency responses and parameters of ”741” OA [1].

OTA (Operational Transconductance Amplifier) [2]

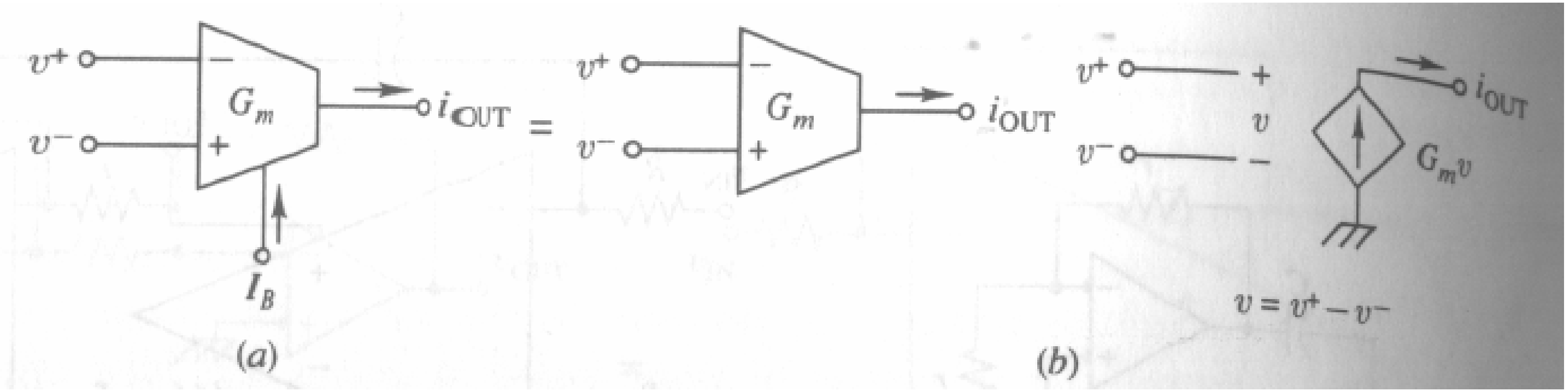


Fig. 9. Typical block diagrams of OTA (a) and its ideal representation (b) [2].

OTA - cont. [2]

Ideal OTA is described by equation:

$$i_{OUT} = G_m(v_+ - v_-)$$

It is often that value of transconductance factor could be controlled by external signal:

$$G_m = G_m(I_B) = \eta I_B$$

Parameters of ideal OTA:

• infinitive input and output resistance,

• transconductance factor $\boldsymbol{G_m}$ of constant value (it does not strive for infinity as it is for ideal OA),

OTA - cont. [2]

Parameters of ideal OTA – cont.:

- voltage gain for OTA without output load tends to infinity,
- no limitation on the ranges of input and output signals,
- infinitive passband,
- full linearity of the amplifier, OTA's linearity is very important in contrast to the OA, it is because OTA works usually without negative feedback,
- no amplification of of common mode signals,
- independence of the output signal on supply voltages, ...

Basic applications of OTAs [2]

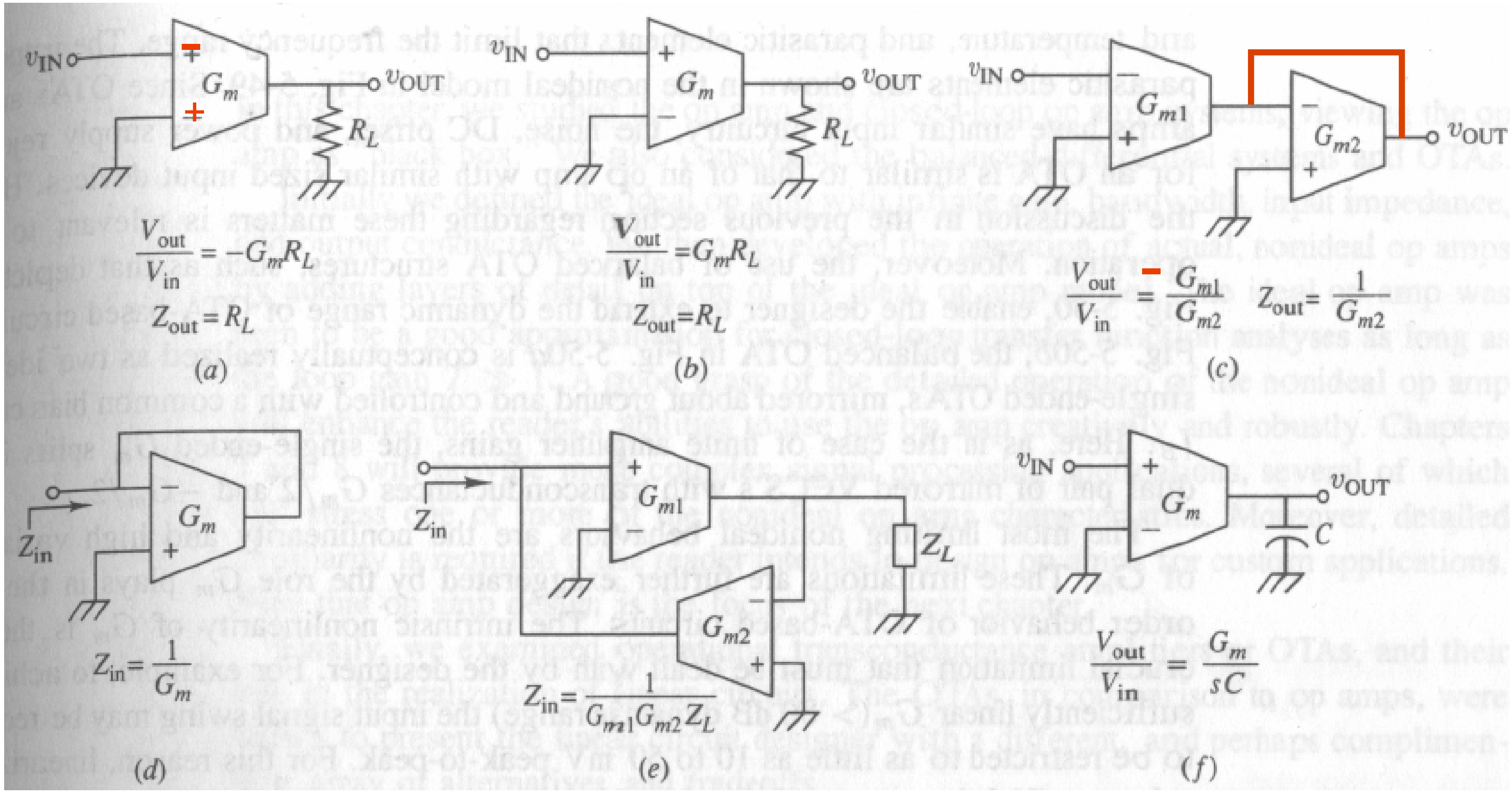


Fig. 10. Basic applications of OTAs (a) inverting amplifier, (b) non-inverting amplifier, (c) inverting amplifier using only OTAs, (d) active impedance, (e) impedance inverter (gyrator), (f) non-inverting integrator [2].

Small-signal OTA's models [1]

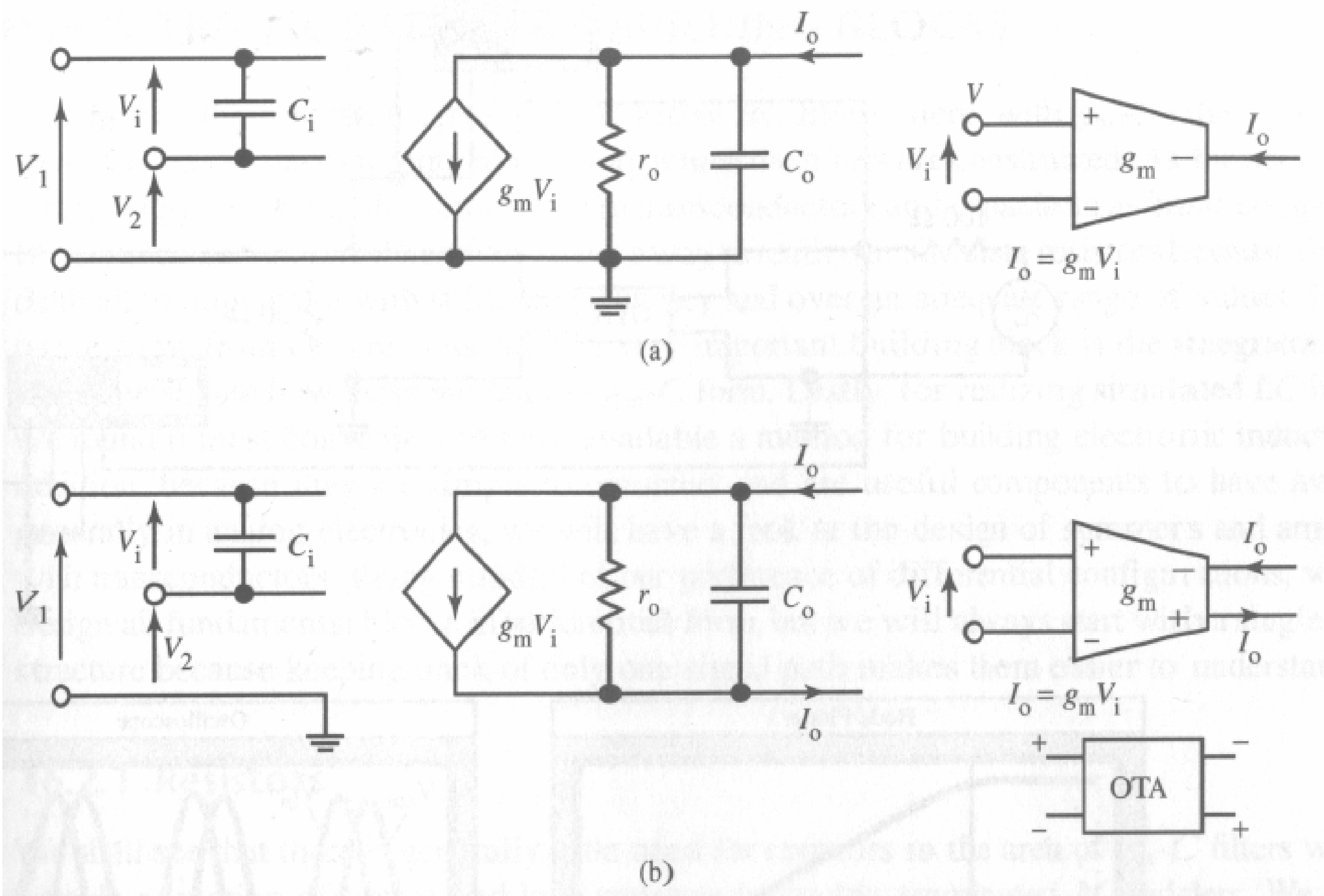


Fig. 11. Small – signal models of single output (a) and fully-balanced (b) OTAs [1].

Why OTAs are suitable for HF and IC ?

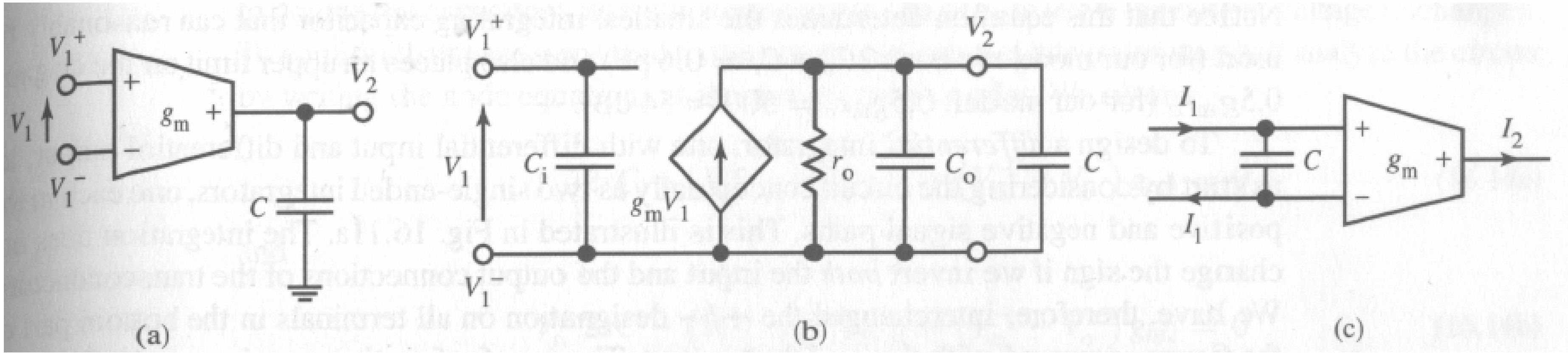


Fig. 12. Voltage mode integrator (a) and its small-signal model (b), current mode integrator (c) [1].

Designed capacitances are connected in parallel to the parasitic capacitancess - so the latter can be included in the design process and almost completely eliminated. Such compensation is called predistortion.

Square - law MOS device model

Basic equations for N MOS transistor

$$V_T = V_{T0} + \gamma(\sqrt{\phi - v_{BS}} - \sqrt{\phi})$$

$$K' = \mu C_{OX}$$

drain current in ohmic (also called triode) region:

$$i_D = \frac{K'W}{L}\left(v_{GS} - V_T - v_{DS}/2\right)v_{DS}$$

$$K = \frac{1}{2}K'\frac{W}{L} = \frac{1}{2}\mu C_{OX}\frac{W}{L}$$

drain current in saturation (also called penthode) region:

$$i_D = \frac{K'W}{2L}\left(v_{GS} - V_T\right)^2\left(1 + \lambda v_{DS}\right)$$

where: V_T – threshold voltage [V],
V_{T0} – threshold voltage for zero bulk-source voltage, i.e. for V_{BS}=0 [V],
γ- bulk effect parameter [$\sqrt{V}$],
ϕ- surface potential (about 0.7V),
μ- mobility of the carriers in the channel [m^2/(sec V)],
C_{OX} – oxide capacitance by unit area [F/m^2],
λ- channel length modulation coefficient [1/V],
W, L – the width and the length of the MOS device channel, respectively [m].

Analysis of a simple OTA circuit

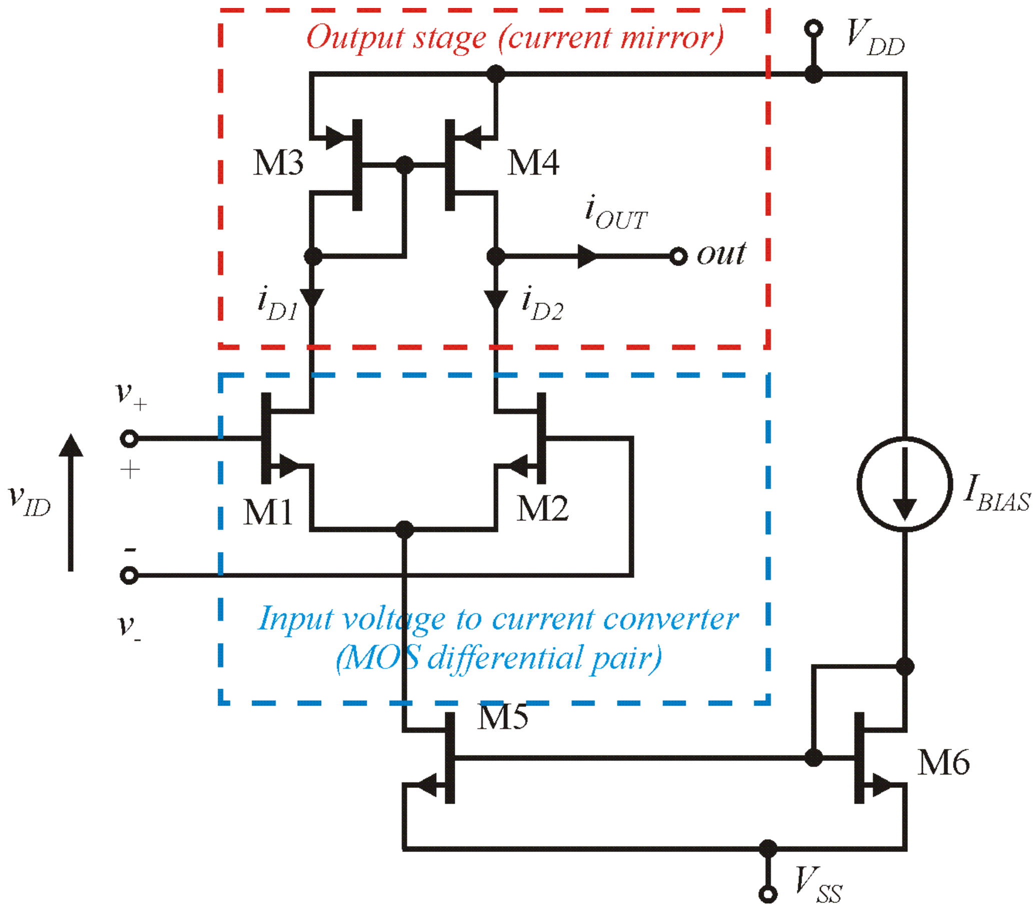


Fig. 13. Simple CMOS OTA.

Assumptions:

- the transistors are identical in pairs, i.e. M1 = M2, M3 = M4 and M5 = M6, which amounts to the equality of their transconductance coefficients K,
- the operating point is determined by the I_{BIAS} current source,
- in the first approximation, the output resistances of the devices are omitted.

Analysis of OTA, I / O voltage range

The operating point w determines the I_{BIAS} current, which gives the following gate - source voltages:

$$V_{GS5} = V_{GS6} = \sqrt{\frac{I_{BIAS}}{K_5}} + V_{TN} \qquad V_{GS1} = V_{GS2} = \sqrt{\frac{1/2 I_{BIAS}}{K_1}} + V_{TN}$$

$$\left|V_{GS3}\right| = \left|V_{GS4}\right| = \sqrt{\frac{1/2 I_{BIAS}}{K_3}} + \left|V_{TP}\right|$$

Common mode positive/negative voltage swing at input of OTA (for zero input differential voltage) assuming all devices in saturation is equal to:

$$V_{SWI-} = V_{SS} + V_{GS5} - V_{TN} + V_{GS1} \qquad V_{SWI+} = V_{DD} - \left|V_{GS3}\right| + V_{TN}$$

Output voltage range assuming all devices working in saturation is equal to:

$$V_{SWO-} = v_{I-} - V_{TN} \qquad V_{SWO+} = V_{DD} - \left|V_{GS4}\right| + \left|V_{TP}\right|$$

Analysis of OTA, simplified small - signal analysis for low frequencies

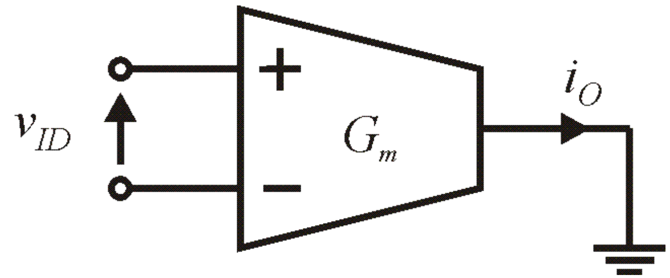


Fig. 14. Small-signal analysis is made assuming short – circuit at output.

For low frequencies:

$$i_0 = i_{d1} - i_{d2} = \frac{1}{2} v_{id}\, gm_1 + \frac{1}{2} v_{id}\, gm_2 \Big|_{gm_1 = gm_2} = v_{id}\, gm_1$$

where:

$$gm_1 = \sqrt{2K_1 I_{BIAS}}$$

Analysis of OTA, simplified small - signal analysis – cont.

Simplified small-signal model for frequency analysis, it takes into account only the gate-source capacitance of the current mirror.

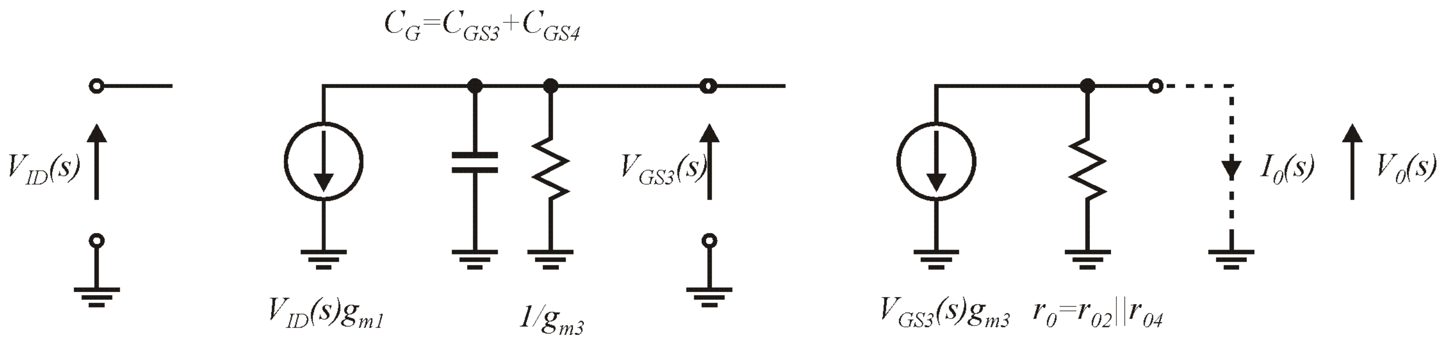


Fig. 15. Simplified small-signal model of OTA from Fig. 13.

Capacitors and resistors values can by expressed by equations:

$$C_G = C_{GS3} + C_{GS4} = \frac{2}{3}\left(W_3 L_3 + W_4 L_4\right) C_{OX} \qquad r_{02} = r_{04} = \frac{1}{\lambda I_{D2}} = \frac{2}{\lambda I_{BIAS}}$$

where: C_{OX} – oxide capacitance by unit area, λ- channel length modulation coefficient .

For the output short circuit as in Fig. 15, the OTA transfer function is equal to:

$$Gm(s) = \frac{I_0(s)}{V_{ID}(s)} = gm_1 \frac{gm_3}{SC_G + gm_3}$$

So transmittance has a parasitic pole for radial frequency equal to:

$$\omega_P = -\frac{gm_3}{C_G}$$

However, the voltage gain when the output is opened will be equal to:

$$\frac{V_0(s)}{V_{ID}(s)} = gm_1 r_0 \frac{gm_3}{SC_G + gm_3}$$

Low frequency voltage gain is therefore equal to:

$$A_0 = gm_1 r_0$$

Summary of OTA small-signal analysis:

• it is possible to regulate the amplifier's transconductance by adjusting the I_{BIAS} current, this control is square root relationship and therefore the resultant transconductance changes are not very large,

• the parasitic pole lies at $\omega=-gm_3/C_G$, which results in severe frequency limitation, the solution may be W/L modification to broaden the band, usage of high frequency current mirror or usage of one-stage OTA with negative load resistance,

• finite low frequency voltage gain may turn out to be too small for certain applications, in such a case you some can apply: large L-size transistors, cascade current mirrors loads or negative resistance loads of the OTA.

Large-signal analysis of a CMOS pair

To simplify the calculations, we introduce the following designations:

$$i_{D1} = I + i \qquad i_{D2} = I - i \qquad I = \frac{I_{BIAS}}{2} \qquad K = K_1 = K_2$$

Kirchhoff voltage law for the input circuit can be written as follows:

$$v_{ID} = v_{GS1} - v_{GS1} = \left[\sqrt{\frac{I+i}{K}} + V_{TN}\right] - \left[\sqrt{\frac{I-i}{K}} + V_{TN}\right]$$

We solve the above equation in order to determine current *i*:

$$i = \sqrt{KI}\, v_{ID} \sqrt{1 - \frac{K v_{ID}^2}{4I}}$$

Large-signal analysis – cont.

The value of the input voltage for which the saturation of the CMOS pair occurs, i.e. $i=I$ is equal to:

$$\left|v_{ID}\right| \leq \sqrt{2}\sqrt{\frac{I}{K}} = V_{IDMAX}$$

One can find the input voltage where the output current i differs (1-E) times from the ideal value (the E factor is the error of the output current):

$$\sqrt{KI}\,v_{ID}\sqrt{1-\frac{Kv_{ID}^2}{4I}} = (1-E)\sqrt{KI}\,v_{ID}$$ so:

$$v_{IDError} = 2\sqrt{\frac{I}{K}}\sqrt{1-(1-E)^2} = \sqrt{2}\sqrt{1-(1-E)^2}\,V_{IDMAX}$$

For example, if E=1%, then:

$$v_{IDError} == \sqrt{2}\sqrt{1-(1-0{,}01)^2}\,V_{IDMAX} \approx 0{,}2V_{IDMAX}$$

Large-signal analysis – cont.

Large - signal OTA transconductance can be determined by calculation of the derivative of the output current in respect to the input voltage:

$$\frac{1}{2}Gm = \frac{di}{dv_{ID}} = \sqrt{KI}\left[\sqrt{1-\frac{Kv_{ID}^2}{4I}} - \frac{\frac{Kv_{ID}^2}{4I}}{\sqrt{1-\frac{Kv_{ID}^2}{4I}}}\right]$$

Similarly as for a current error, a transconductance error can be determined. A 1% transconductance error occurs for approximately $0{,}11V_{IDMAX}$.

Large-signal analysis – cont.

Large - signal characteristics for K=100μA/V^2, V_{TN}=1V, I=50 μA

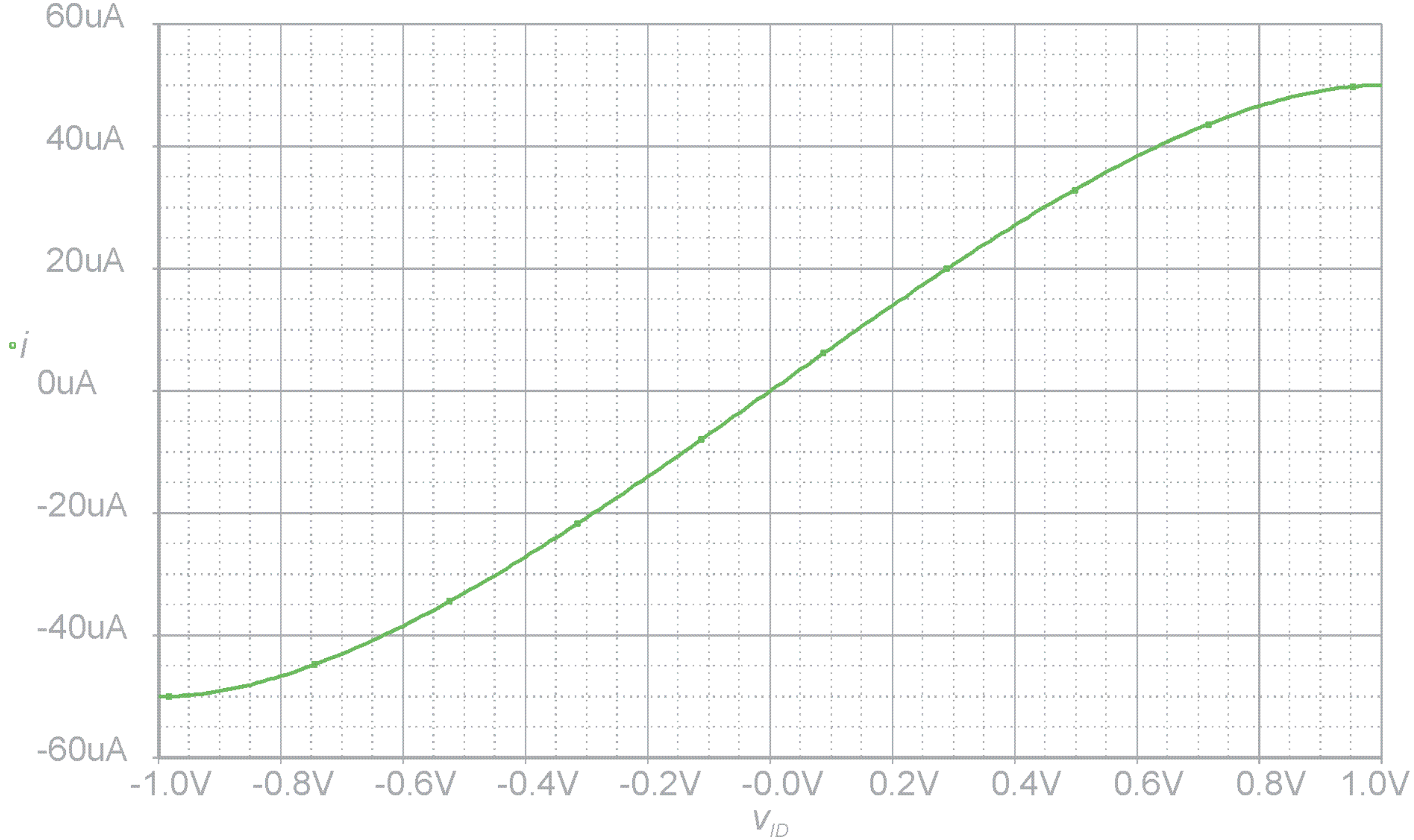


Fig. 16. Output current of the MOS differential pair from Fig. 13.

Large-signal analysis – cont.

Large - signal characteristics for K=100μA/V^2, V_{TN}=1V, I=50 μA

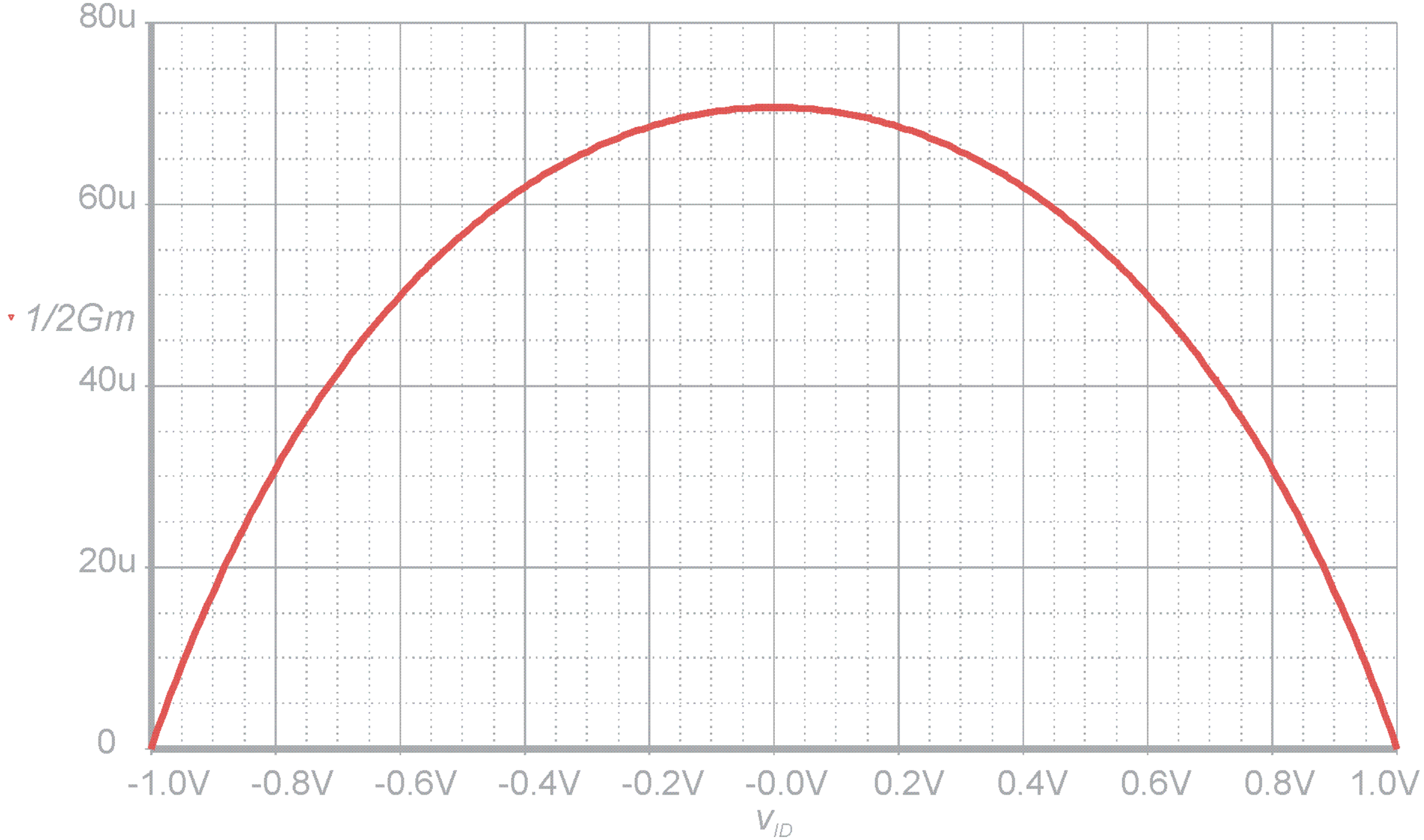
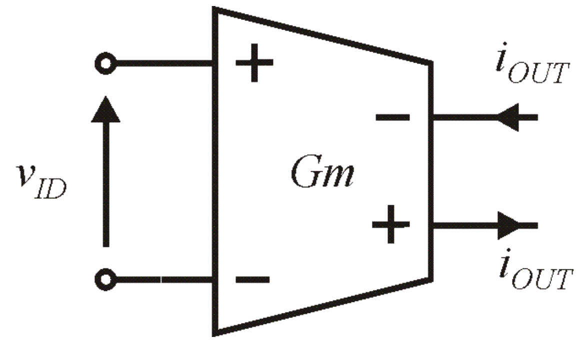


Fig. 17. Large – signal transconductance of MOS pair from Fig. 13.

Large-signal analysis – summary:

- the differential CMOS pair has a small range of linearity of the input differential voltage, representing about 20% of the entire available input signal for 1% of the current error, alternatively representing about 11% of the possible input signal for 1% of the transconductance error,
- in order to increase the linearity range, a different V-I converter should be used (eg MOS pairs in cross connection, degenerating resistors, four transistors in cross connection, MOS transistors in the triode range, asymmetrical pairs and others),
- in the differential CMOS pair, tuning of the transconductance Gm value can be implemented by adjusting the I_{BIAS} current, the change is the square root function of this current.

Fully differential OTA



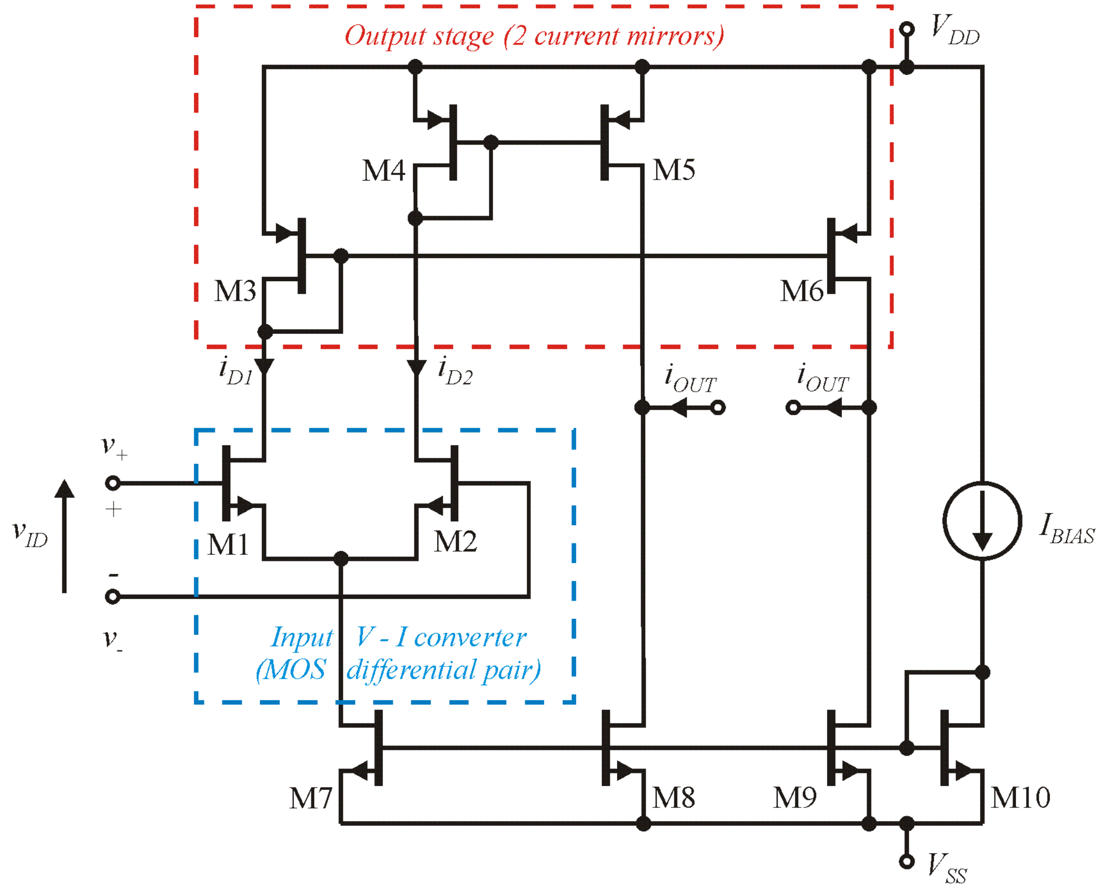


Fig. 18. Fully differential OTA, block diagram and circuit realization with the use of MOS differential pair and current mirrors.

Common Mode Feedback (CMFB)

In an amplifier with a fully differential output, in the case of a differential load or a capacitive load, there is the problem of self-acting appearance of voltage on individual outputs in respect to the ground. Therefore, these amplifiers have to be equipped with a circuit stabilizing common – mode component of the output voltage. Such a citcuit is called Common Mode Feed Back (CMFB).

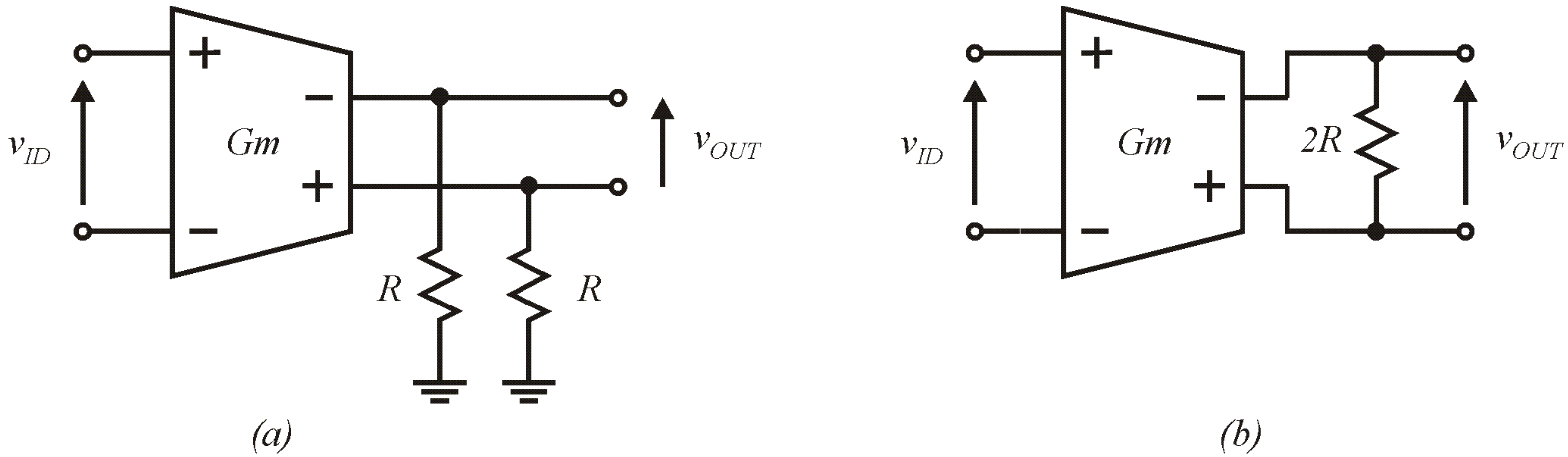
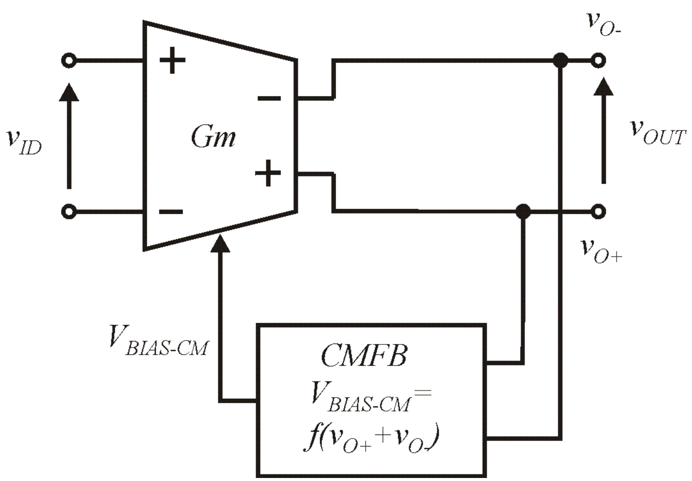
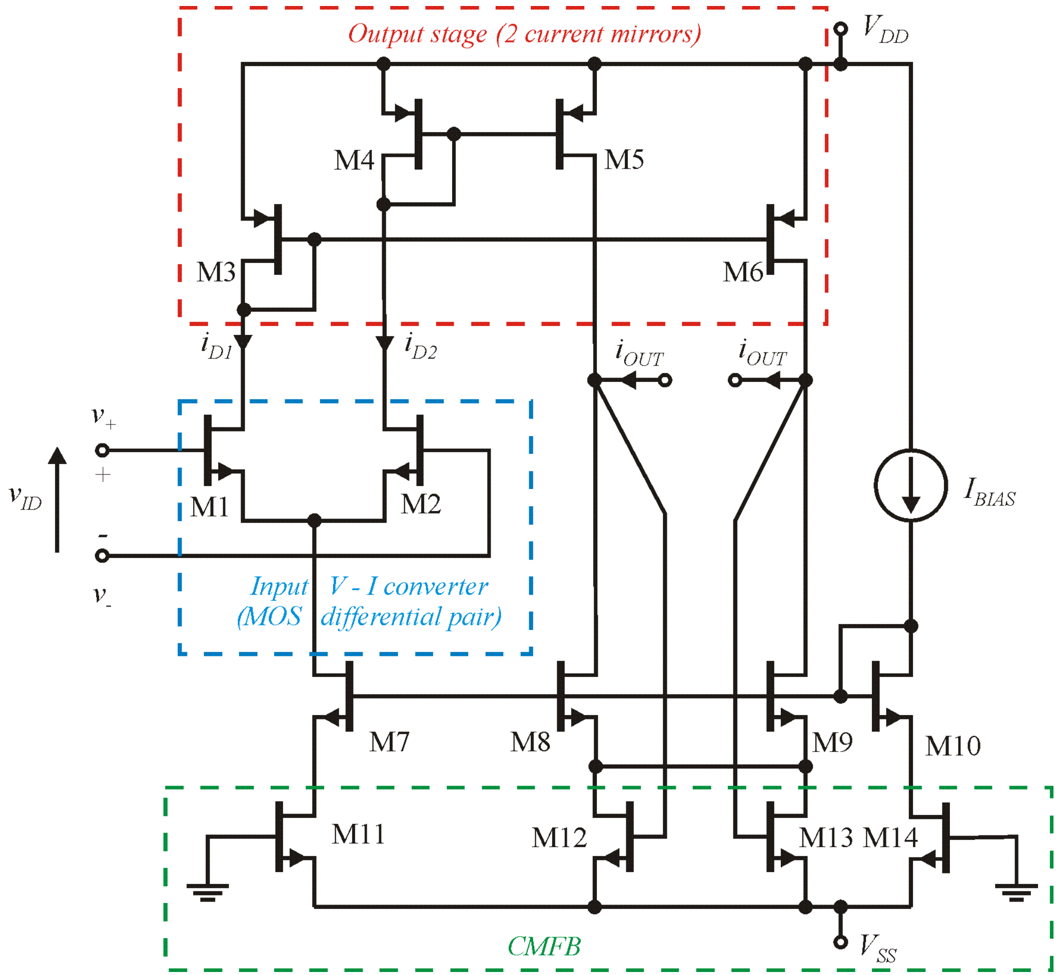


Fig. 19. Fully differential OTA loaded in respect to the ground (a) and fully differentially (b). In case (b) it have to be equipped with CMFB, otherwise output stage of the amplifier will exhibit high value of common-mode voltage and output stage will saturate.

CMFB example of realization



(a)



(b)

Fig. 20. Principles of operation of CMFB circuitry (a) and realization utilizing MOS devices working in triode region (b).

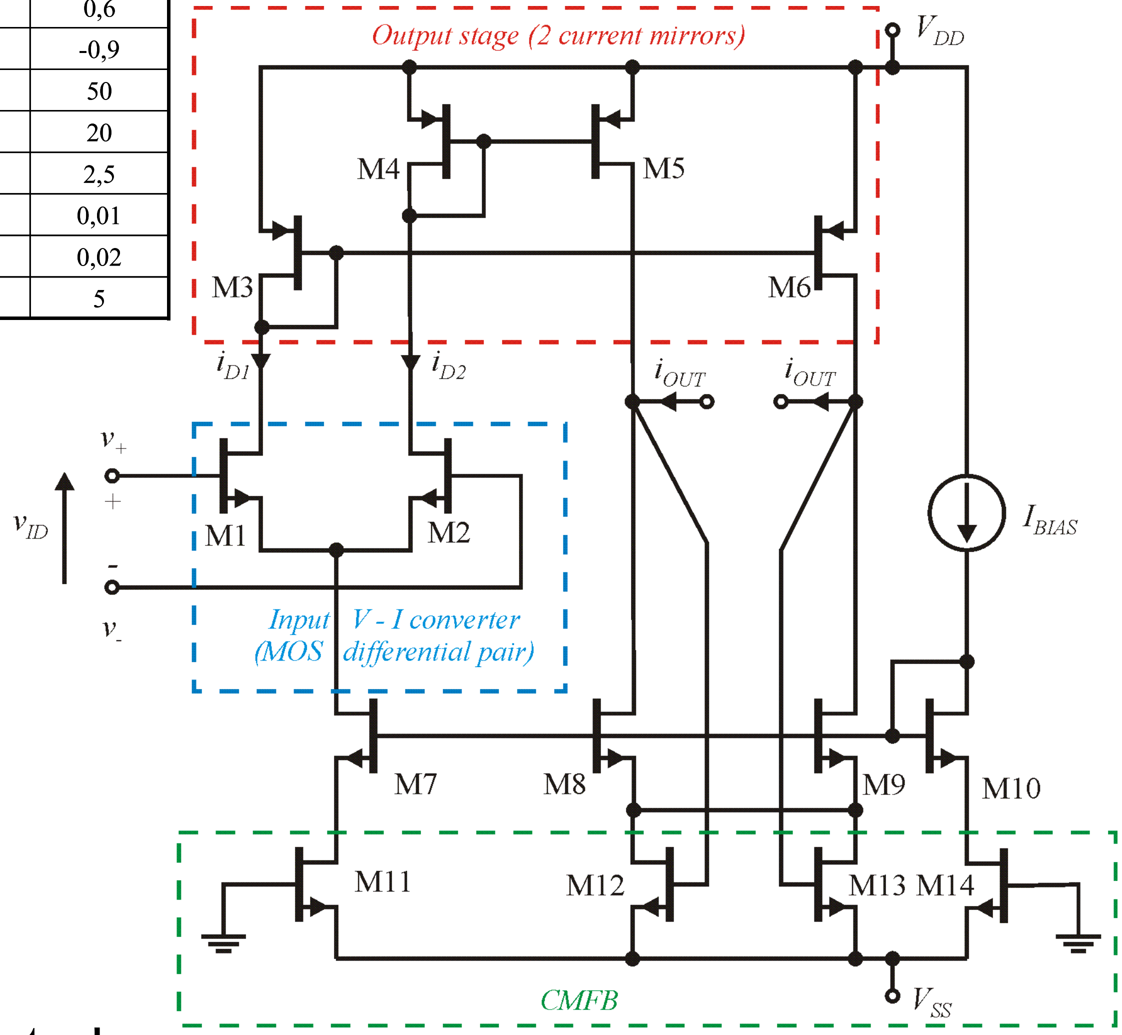
Computational examples: parameters for typical 0.5μm CMOS process

L.p.	Parametr	Unit	Value
1	V_{TN}	[V]	0,6
2	V_{TP}	[V]	-0,9
3	$0,5\mu_N C_{OX}$	[μA/V^2]	50
4	$0,5\mu_P C_{OX}$	[μA/V^2]	20
5	C_{OX}	[fF/μm^2]	2,5
6	λ_N	[1/V]	0,01
7	λ_P	[1/V]	0,02
8	V_{DD}-V_{SS}	[V]	5

Calculation example no 1: OTA

Device	Dim. W/L [µm/µm]
M1-M4	**4/4**
M5, M6	**16/4**
M7	**8/5**
M8, M9	**16/5**
M10	**4/5**
M11	**4/5**
M12, M13	**8/5**
M14	**2/5**

L.p.	Parametr	Unit	Value
1	V_{TN}	[V]	0,6
2	V_{TP}	[V]	-0,9
3	$0,5\mu_N C_{OX}$	[µA/V²]	50
4	$0,5\mu_P C_{OX}$	[µA/V²]	20
5	C_{OX}	[fF/µm²]	2,5
6	λ_N	[1/V]	0,01
7	λ_P	[1/V]	0,02
8	V_{DD}-V_{SS}	[V]	5



Parameter	Value
V_{DD}	**2,5V**
V_{SS}	**-2,5V**
V_{BIAS}	**10µA**

All previously presented parameters of the OTA have to be calculated (pages 39-46).

Calculation example no 1: OTA – cont.

The results of the calculations are as follows:

I_{D10}=10μA, I_{D7}=20μA, I_{D8}= I_{D9}= 40μA, I_{D5}= I_{D6}= 40μA,

K_1= 50μA/V^2, K_3= 20μA/V^2, K_{10}= 40μA/V^2, K_{14}= 20μA/V^2,

V_{GS1}=1,047V, $|V_{GS3}|$=1,607V, $V_{GS7..GS8}$=1,1V, $V_{DS11..DS14}$=0,132V,

V_{SWI-}=-0,821V, V_{SWI+}=1,493V, V_{SWO-}=-1,868V, V_{SWO+}=1,907V,

Gm=89,44μS, ω_P=-212,2Mrad/sec (33,77MHz),

Ao=149,1V/V=43,5dB, V_{IDMAX}=0,632V,

1% current error occurs for $|v_{ID}|$=0,126V,

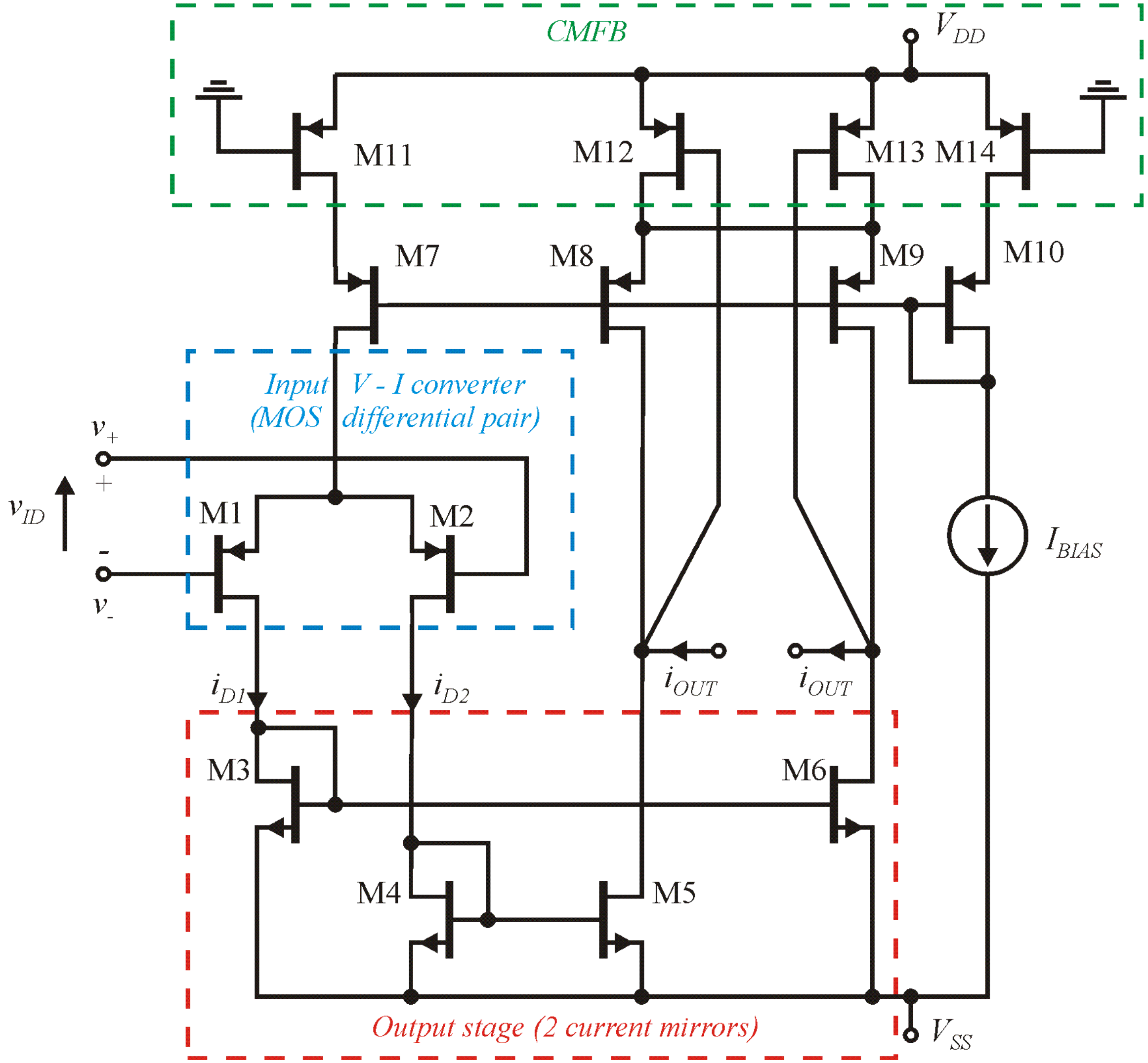
1% transconductance *Gm* error occurs for $|v_{ID}|$=0,0695V,

Homework no 1

For the OTA amplifier shown in the drawing, determine the parameters discussed earlier in the lecture. Technology parameters as previously stated.

Device	Dimmensions W/L [μm/μm]
M1,M2	**12/4**
M3, M4	**2/2**
M5,M6	**4 * 2/2**
M7	**2 * 4/2**
M8, M9	**4 * 4/2**
M10	**4/2**
M11	**2 * 2/2**
M12, M13	**4 * 2/2**
M14	**2/2**

Parameter	Value
V_{DD}	**2,5V**
V_{SS}	**-2,5V**
I_{BIAS}	**10μA**



First and second generation current conveyor

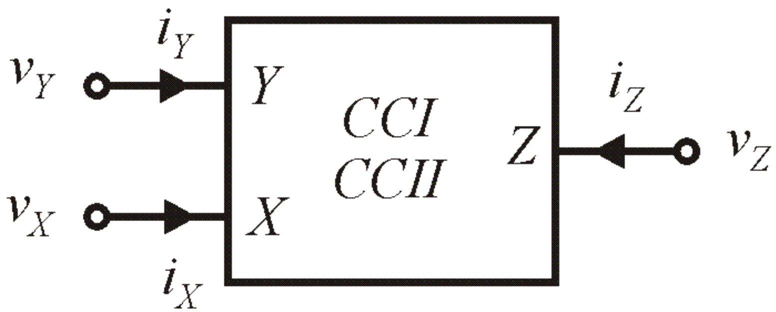


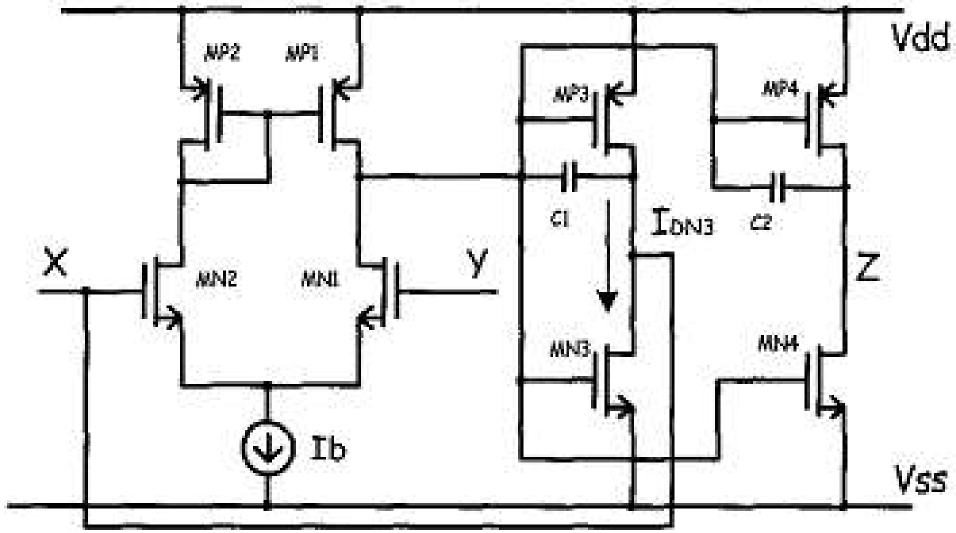
Fig. 21. Graphical representation of current conveyor. CCI stands for first generation current conveyor while CCII is for second generation

$$\begin{bmatrix} i_Y \\ v_X \\ i_Z \end{bmatrix} = \begin{bmatrix} 0 & a & 0 \\ 1 & 0 & 0 \\ 0 & \pm 1 & 0 \end{bmatrix} \begin{bmatrix} v_Y \\ i_X \\ v_Z \end{bmatrix}$$

For CCI a=1, while for CCII a=0. For a positive conveyor, the "+" sign next to the one is selected.

In the literature there are also known conveyors with current amplification, in such a case "1" from the bottom row changes into the current gain factor "k".

CCII+ exemplary internal structure and parameters [3]

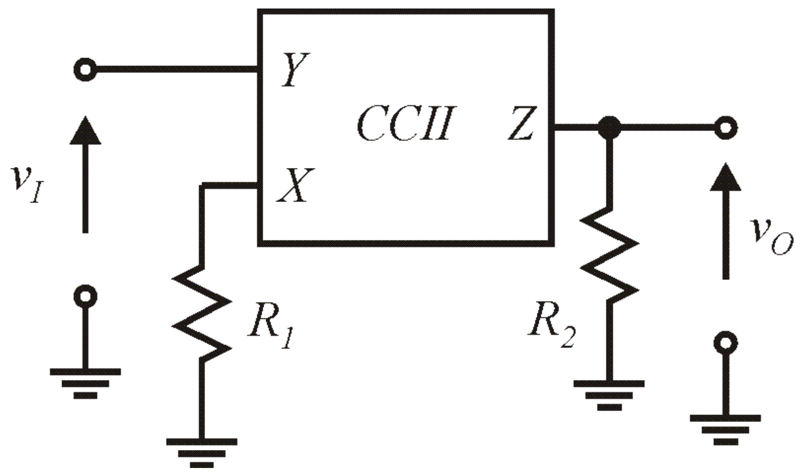


Characteristics		
V_{DD}-V_{SS}	**1.5 V**	**1.2 V**
I_{DN3} (see fig.1)	68µA	2.68 µA
Rx	9.98 Ω	192 Ω
Cy	0.18 pF	0.18 pF
Rz	125 KΩ	600 KΩ
Cz	0.3 pF	0.8 pF
-3dB Bandwidth	32 MHz	21 MHz
Power Consumption	250 µW	13 µW

(a) (b)

Fig. 22. CCII+ schematic in CMOS technology (a) and its real parameters (b) [3].

Basic applications of CCII - voltage amplifier



$$\frac{v_O}{v_I} = \frac{R_2}{R_1}$$

Fig. 23. Voltage amplifier using CCII+.

Basic applications of CCII - differential voltage amplifier

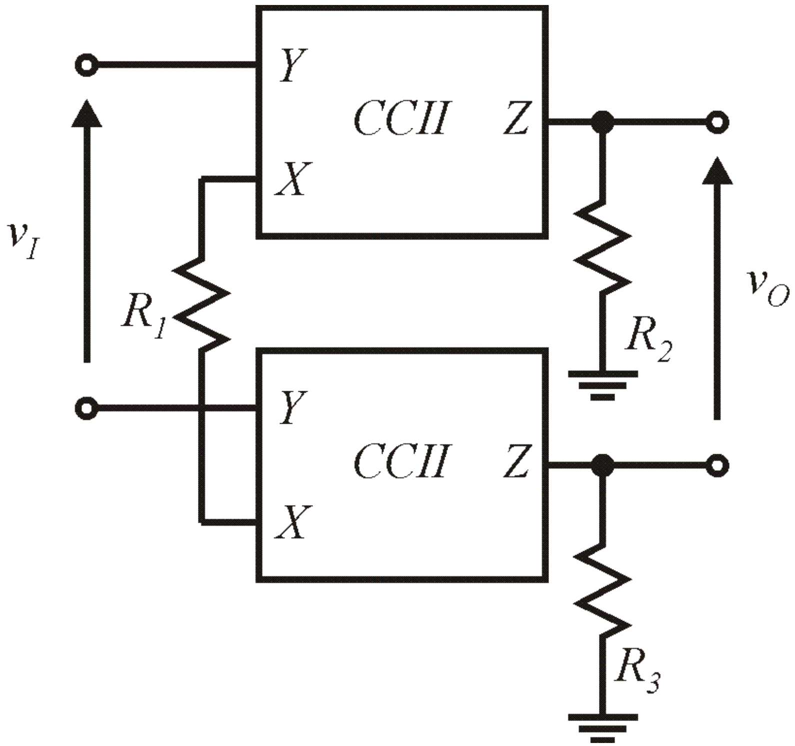
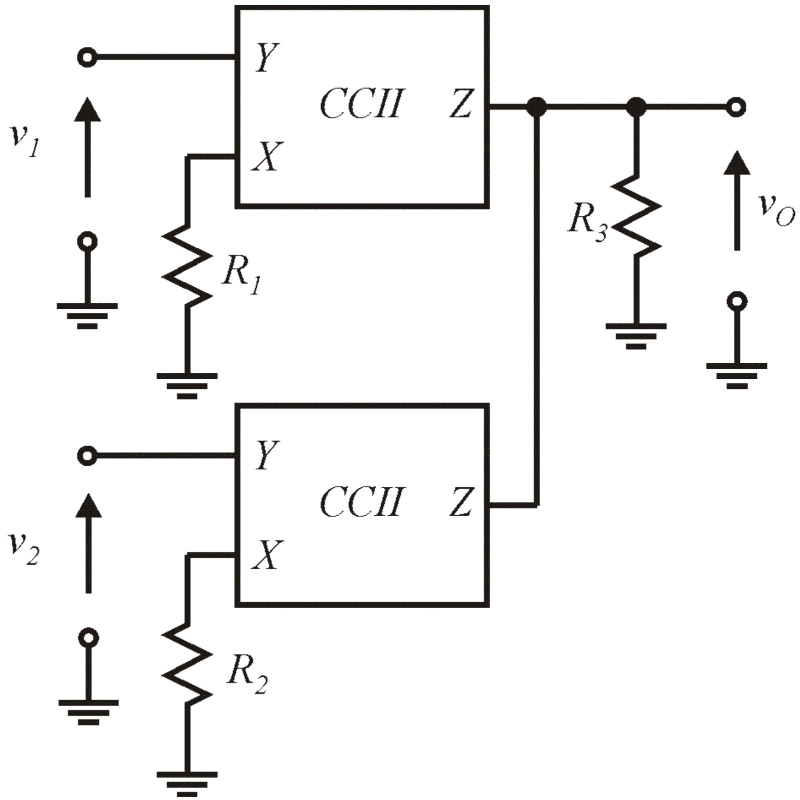


Fig. 24. Differential voltage amplifier using CCII+.

$$\frac{v_O}{v_I} = \frac{R_2 + R_3}{R_1}$$

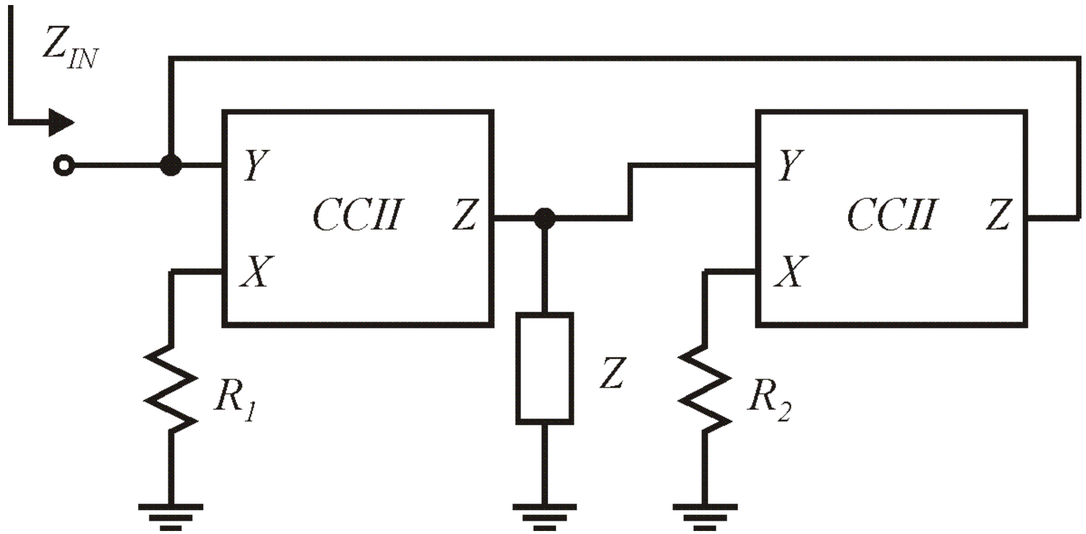
Basic applications of CCII - adder



$$v_O = \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} \right) R_3$$

Fig. 25. Voltage adder with the use of CCII+.

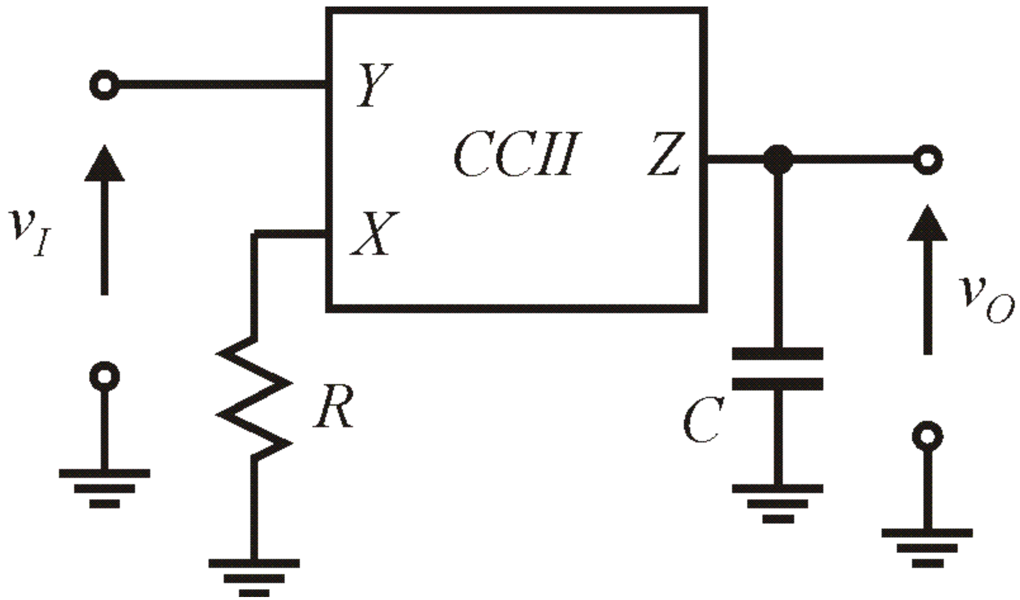
Basic applications of CCII - gyrator



$$Z_{IN} = -\frac{R_1 R_2}{Z}$$

Fig. 26. Gyrator with the use of CCII+.

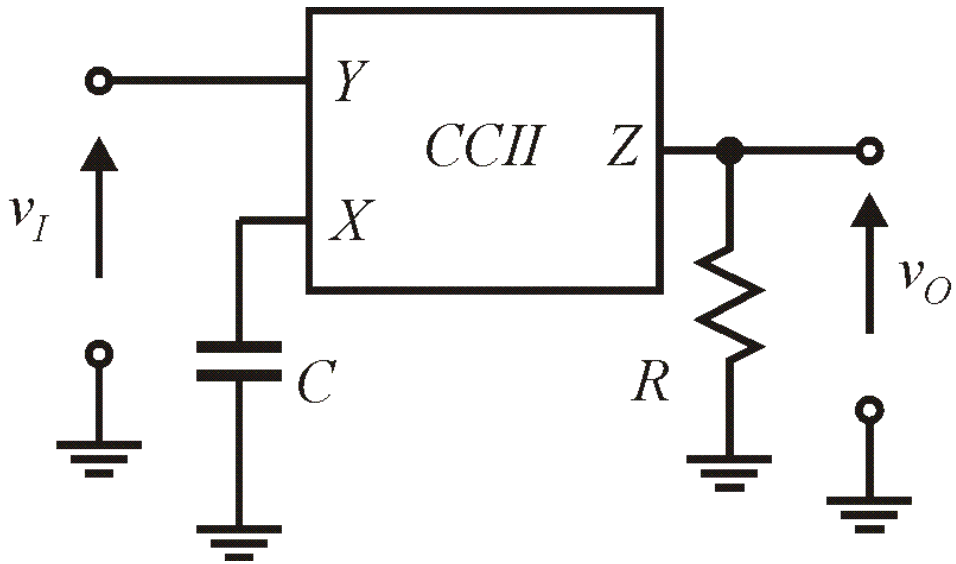
Basic applications of CCII - integrator



$$v_O(t) = \frac{1}{RC}\int_0^t v_I(\tau)d\tau + V_0$$

Fig. 27. Integrator with the use of CCII+.

Basic applications of CCII - differentiating circuit



$$v_O(t) = RC\frac{dv_I(t)}{dt}$$

Fig. 28. Differentiating circuit with the use of CCII+.

Second order sections (biquads), lowpass (LP) section [1]

Transfer function of II-order lowpass section is given by:

$$T(s) = \frac{N(s)}{D(s)} = \pm H \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

where: H – gain for low frequencies, Q – quality factor and ω_0– pole frequency, also sometimes called natural frequency or corner frequency.

Poles of *T(s)* are a pair of conjugates on the left s-plane. Hence if we denote the denominator of *T(s)* as:

$$D(s) = (s + \alpha + j\beta)(s + \alpha - j\beta) = s^2 + 2\alpha s + \left(\alpha^2 + \beta^2\right)$$

Parameters and poles locations can be tied by equations:

Second order sections, LP – cont. [1]

$$\alpha = \frac{\omega_0}{2Q} \qquad Q = \frac{\omega_0}{2\alpha} \qquad \omega_0^2 = \alpha^2 + \beta^2 \qquad \beta = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

$$\psi = \cos^{-1}\left(\frac{\alpha}{\omega_0}\right) = \cos^{-1}\left(\frac{1}{2Q}\right)$$

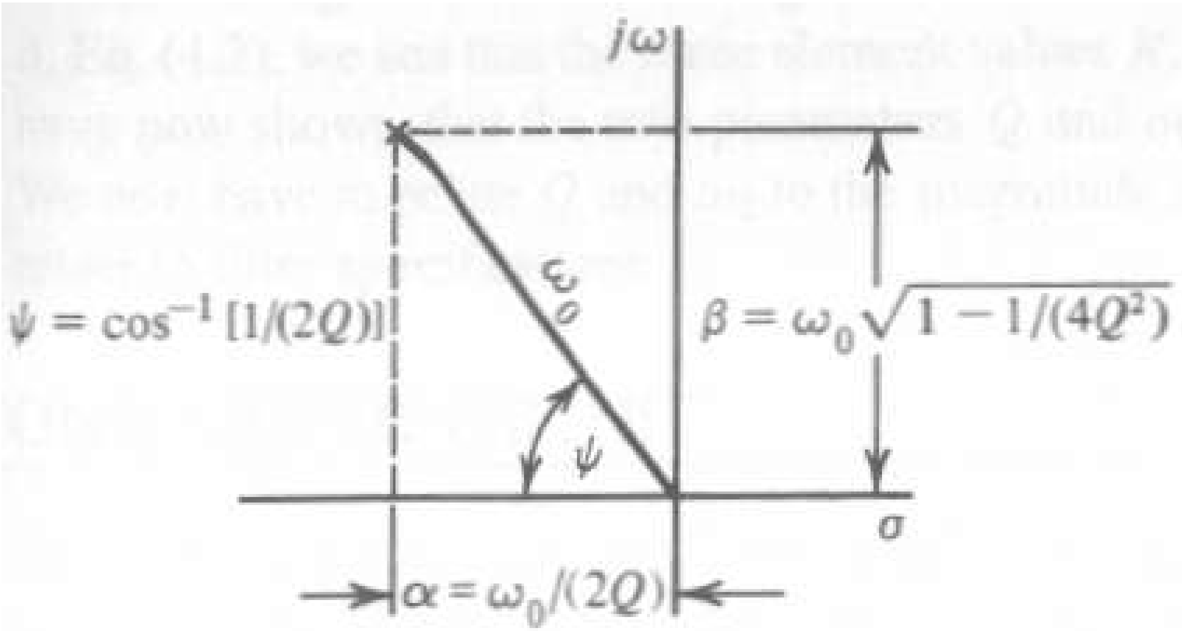


Fig. 29. Pole location and relationship with biquad parameters [1].

Second order sections, LP – cont. [1]

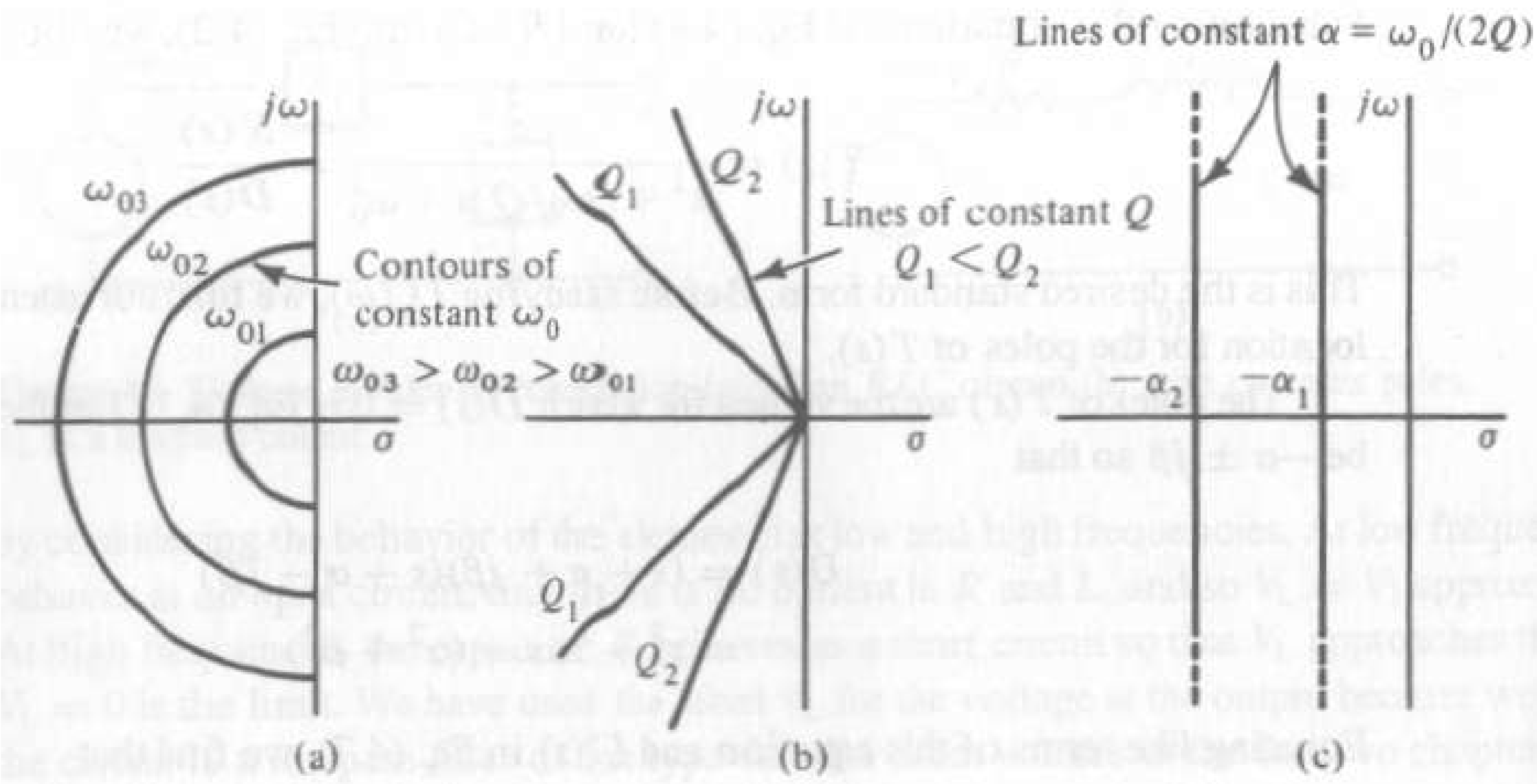


Fig. 30. Pole locations for different values of biquad parameters [1].

Second order sections, LP – cont. [1]

Normalized frequency responses (in respect to module and natural frequency) can be obtained by assumption H=1 and substitution $s_n = s/\omega_0$. Then we get:

$$T(s) = \frac{1}{s^2 + \frac{1}{Q}s + 1} \qquad T(j\omega) = \frac{1}{(j\omega)^2 + \frac{1}{Q}j\omega + 1} = \frac{1}{\frac{j\omega}{Q} + 1 - \omega^2}$$

Thus, the amplitude and phase characteristics are equal to:

$$\left|T(j\omega)\right| = \frac{1}{\sqrt{(\omega/Q)^2 + \left(1-\omega^2\right)^2}} \qquad \theta = -\tan^{-1}\left(\frac{\omega/Q}{1-\omega^2}\right)$$

Second order sections, LP – cont. [1]

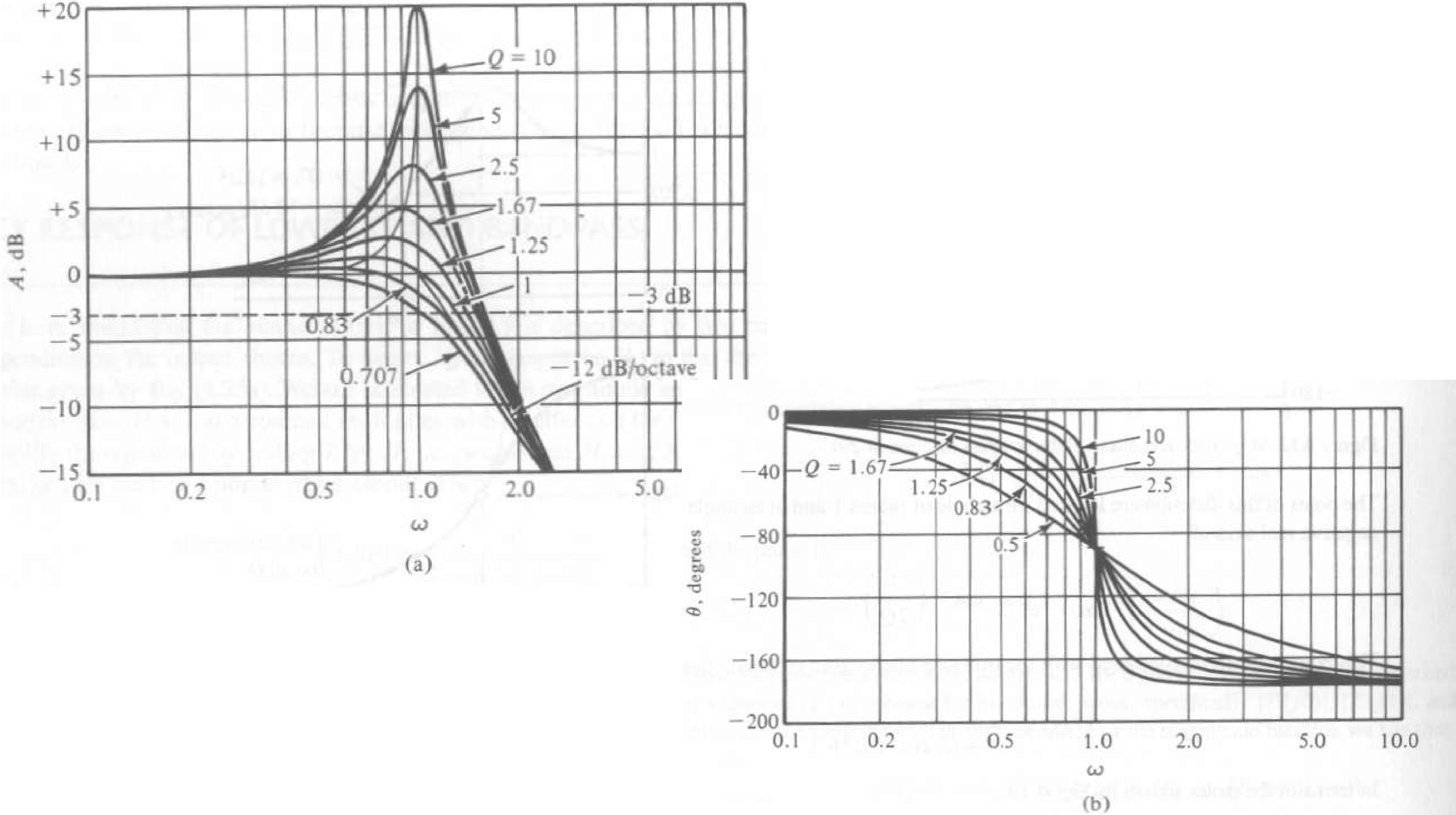


Fig. 31. Frequency responses of the second order lowpass section, (a) amplitude and (b) phase[1].

Second order sections, LP, realization I (sum products)

Physical implementation of the normalized second order section:

$$T(s) = \frac{V_0(s)}{V_I(s)} = \frac{H}{s^2 + \frac{1}{Q}s + 1} \quad \Rightarrow \quad HV_I(s) = V_0(s)\left(s^2 + \frac{1}{Q}s + 1\right) \quad \Rightarrow$$

$$V_0(s) = H\frac{V_I(s)}{s^2} - \frac{V_0(s)}{sQ} - \frac{V_0(s)}{s^2}$$

=>

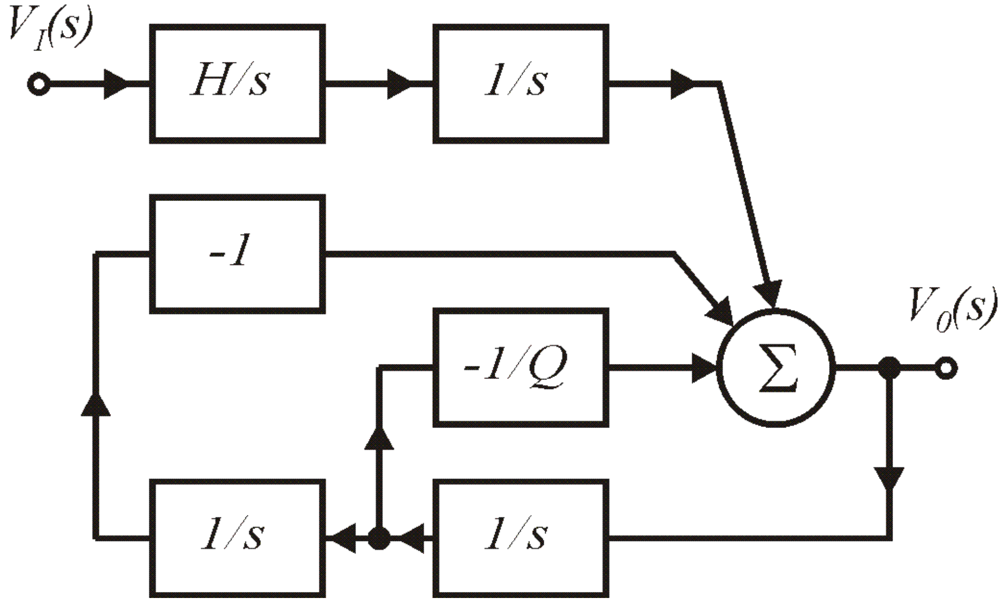


Fig. 32. Direct implementation of the second order lowpass section using products of sum.

II - order sections, LP, realization II (product realization)

$$T(s)=\frac{V_0(s)}{V_I(s)}=\frac{H}{s^2+\frac{1}{Q}s+1} \quad => \quad HV_I(s)=V_0(s)\left(s^2+\frac{1}{Q}s+1\right) \quad =>$$

$$s\left(s+\frac{1}{Q}\right)V_0(s)=HV_I(s)-V_0(s) \quad => \quad V_0(s)=\left(HV_I(s)-V_0(s)\right)\frac{1}{s}\frac{1}{\left(s+1/Q\right)} \quad =>$$

$$V_C(s)=V_B(s)\frac{1}{s}=\frac{1}{s}\left(V_A(s)-\frac{1}{Q}V_C(s)\right) => \frac{V_C(s)}{V_A(s)}=\frac{1}{s+1/Q}$$

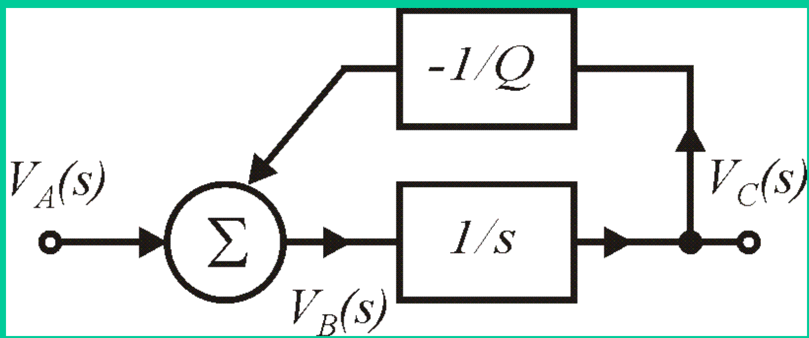


Fig. 33. Realization of block 1/(*s*+*1/Q*) – lossy integrator.

II - order sections, LP, realization II (product realization)

$$V_0(s) = \left(HV_I(s) - V_0(s)\right)\frac{1}{s}\frac{1}{\left(s+1/Q\right)}$$

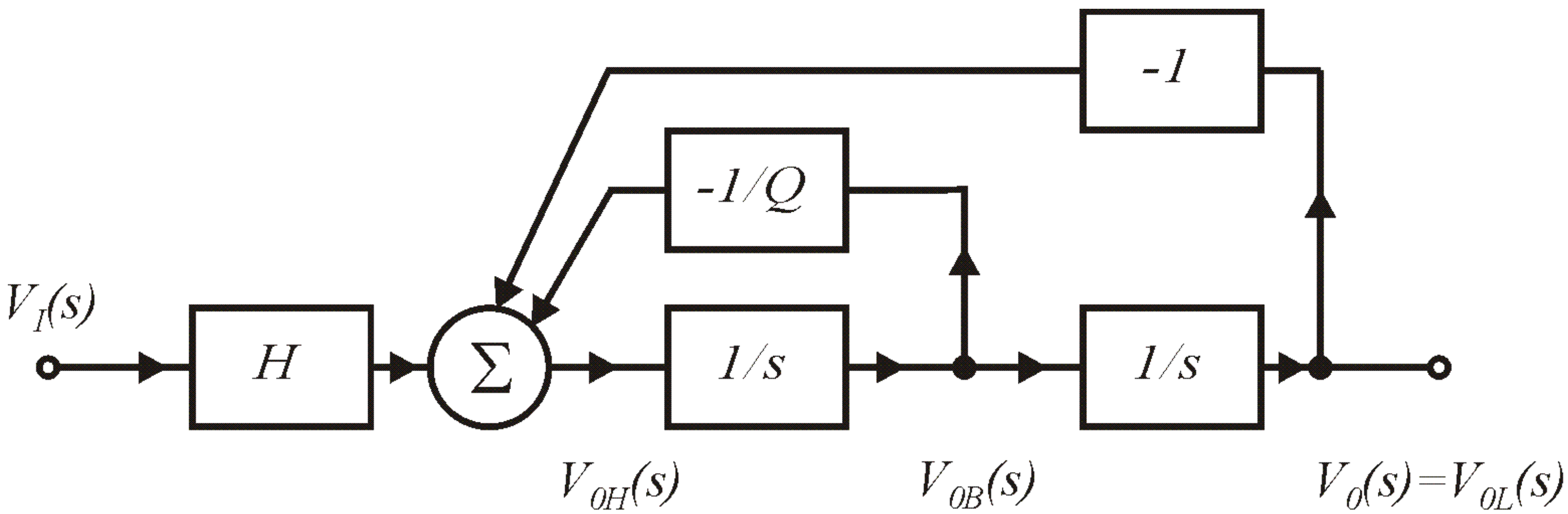


Fig. 34. Direct implementation of the second order lowpass section - products.

Physical implementation of the second-order lowpass section [1]

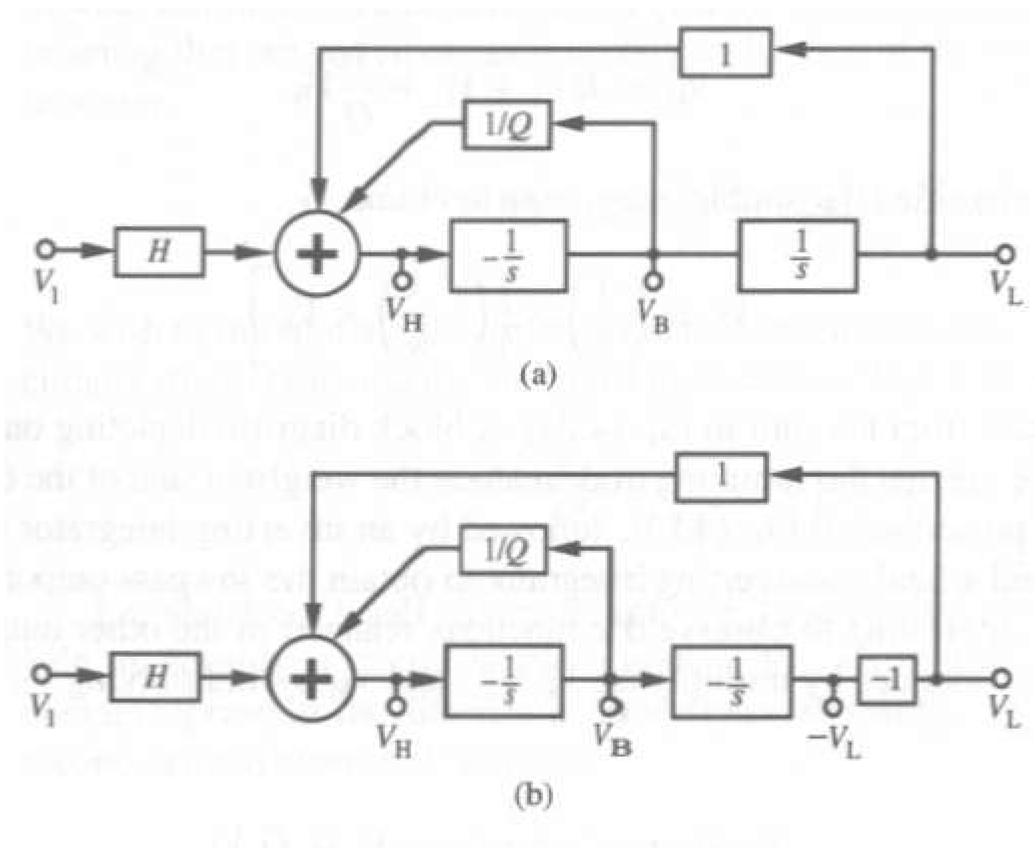


Fig. 35. Realizations of the biquad from Fig. 34 with changes of integration and gain signs [1].

Physical implementation of the second-order lowpass section –cont.[1]

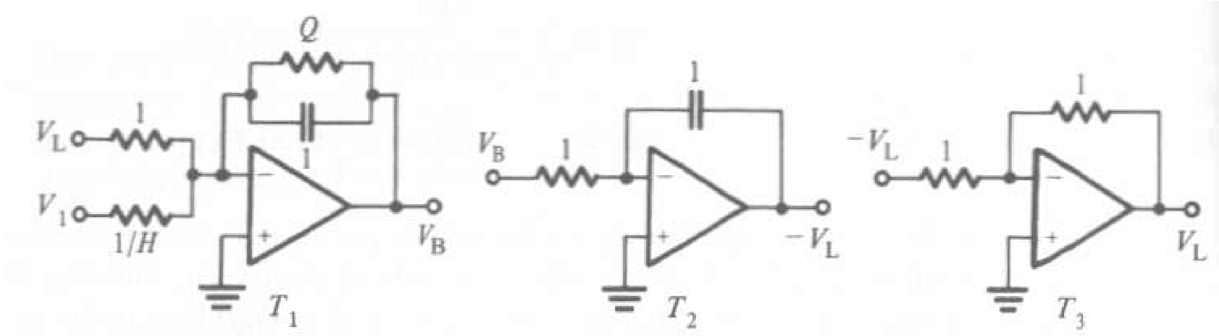


Fig. 36. Components for biquad implementation using operational amplifiers [1].

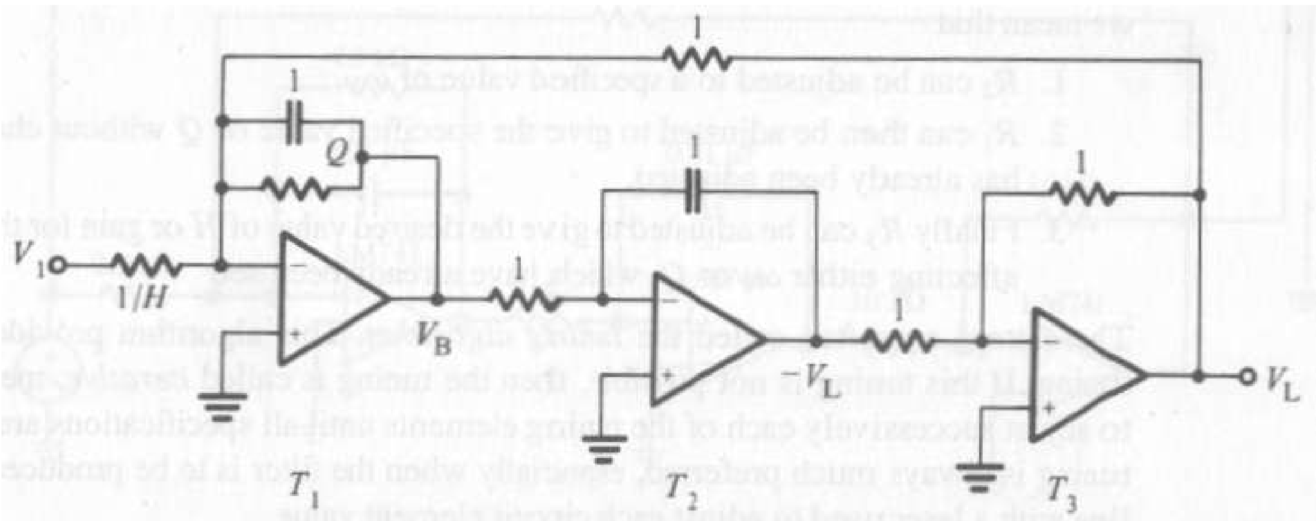


Fig. 37. Tow-Thomas biquad section , normalized [1].

How to get a real filter from the normalized version?

- through denormalization procedure,
- through comparison of circuit having real values components with desired transfer function

Denormalization with the use of calculation example (no 2)

Calculation example: Please design Thow-Thomas lowpass biquad section with natural frequency of value f_0=10kHz, quality factor Q=10 and gain equal to H=1.

Denormalizing radial frequency: $\omega_S=2\pi f_0$=2*3,1415*10kHz= 62831,5[rad/sec],

We assume normalizing resistance of convenient value, for example R_S=10kΩ. Hence all resistors have values of 10k except resistor in lossy integrator whose vale is equal to R_S *Q=100kΩ.

Capacitance values are equal to:
$C=C_N*1/(R_S*\omega_S)$=1/10kΩ/62381,5rad/sec=1,5915nF

Frequency responses after denormalization procedure

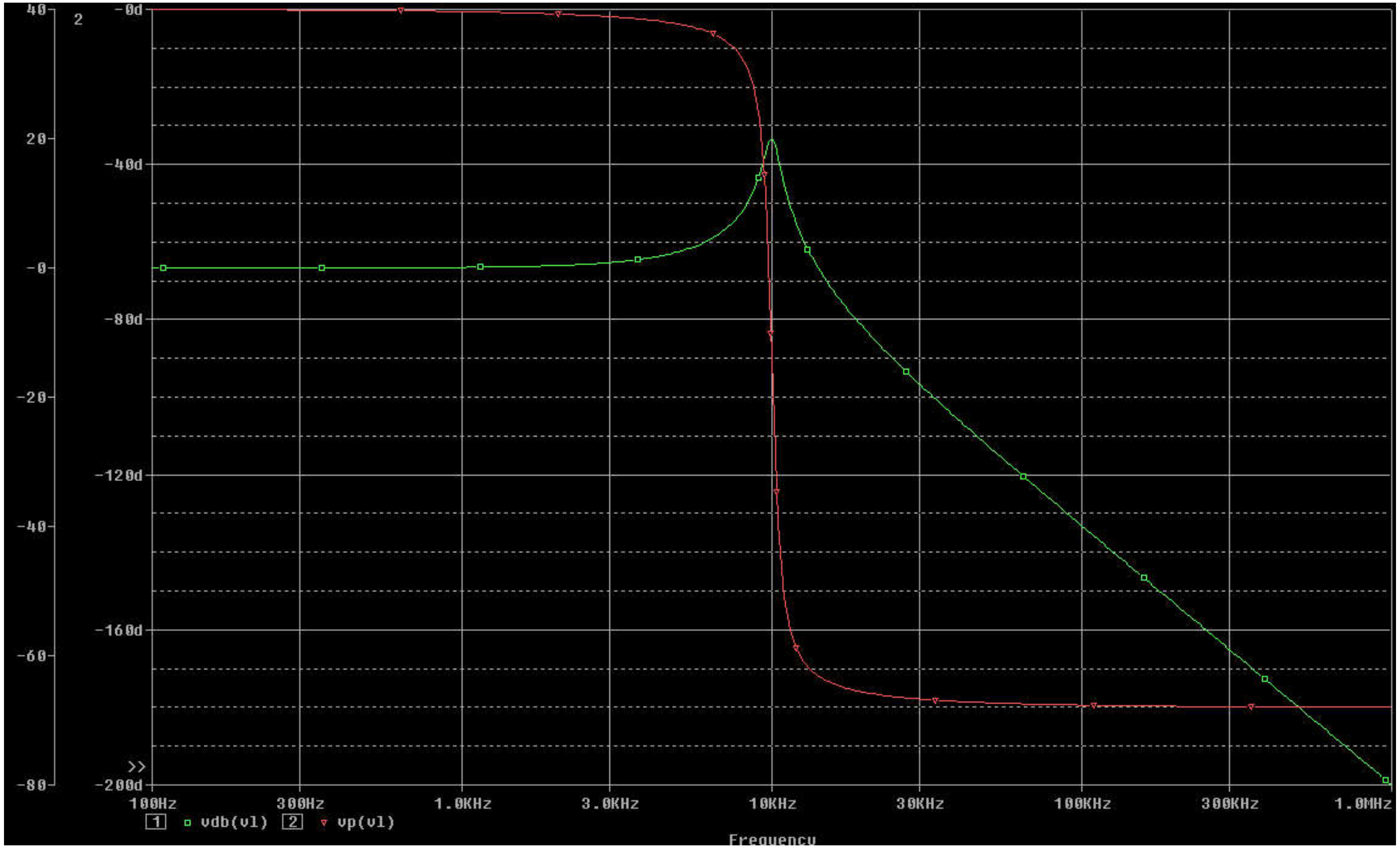


Fig. 38. Frequency responses of Tow-Thomas biquad after denormalization.

Transfer function comparison method [1]

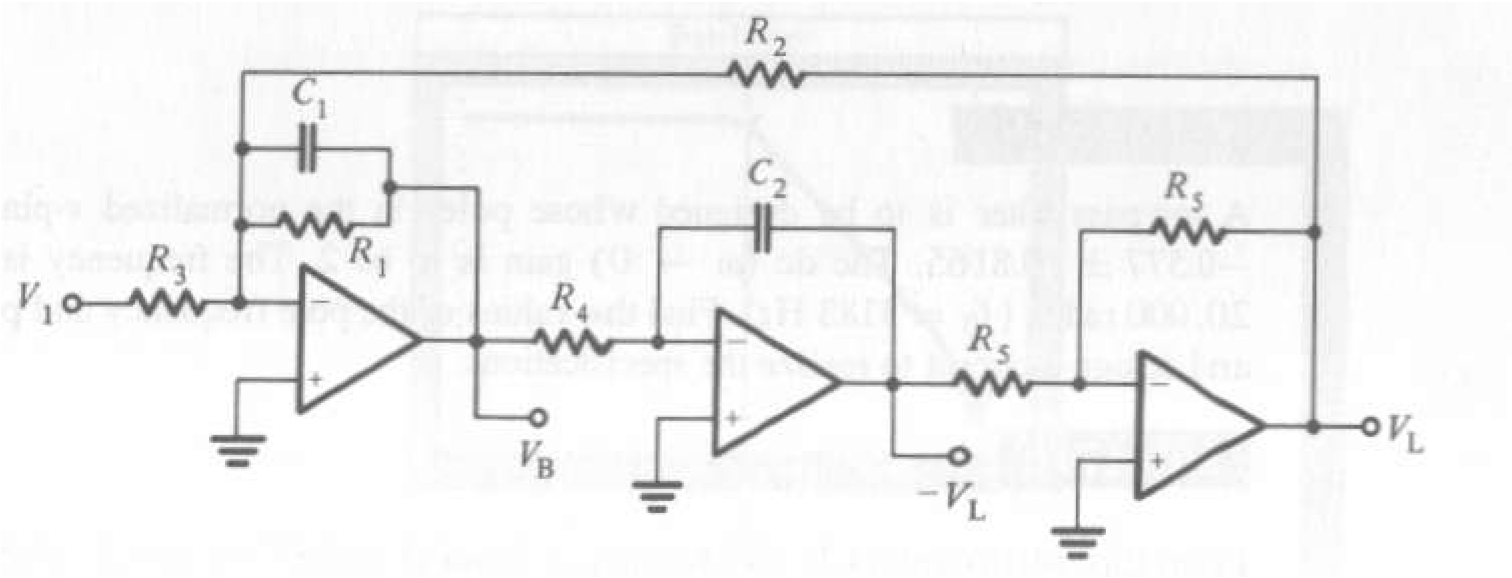


Fig. 39. Tow-Thomas biquad section [1].

Transfer function of the circuit in Fig. 39 is equal to [1]:

$$T(s) = \frac{V_L(s)}{V_I(s)} = -\frac{1/\left(R_3 R_4 C_1 C_2\right)}{s^2 + s/\left(R_1 C_1\right) + 1/\left(R_2 R_4 C_1 C_2\right)}$$

Comparison method – cont. [1]

Transfer function of second – order lowpass section is equal to:

$$T(s) = \pm H \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

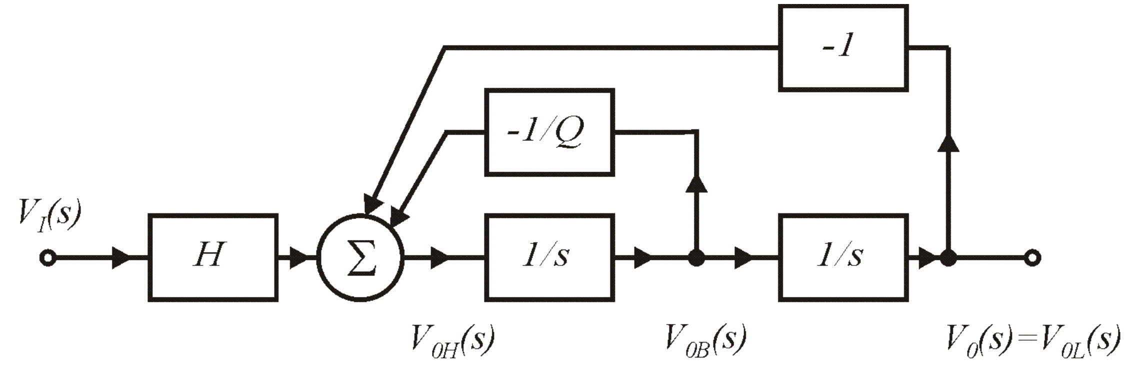
Comparing above equation with transfer function for Tow-Thomas biquad one can obtain:

$$\omega_0^2 = \frac{1}{R_2 R_4 C_1 C_2} \qquad Q = \frac{R_1}{\sqrt{R_2 R_4}} \sqrt{\frac{C_1}{C_2}} \qquad H = \frac{R_2}{R_3}$$

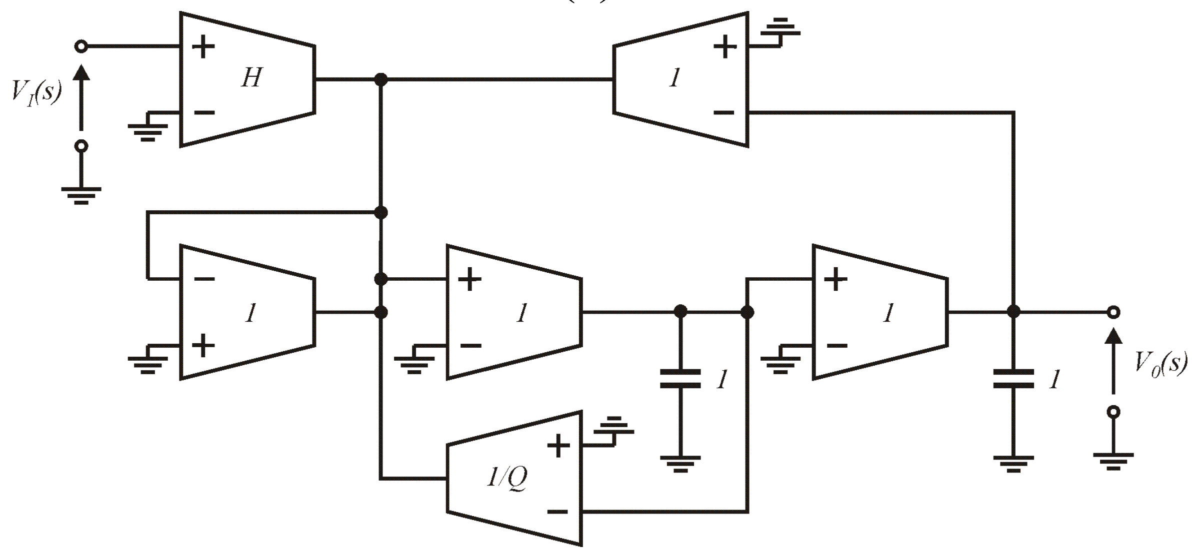
The Tow-Thomas section enables orthogonal (independent) tuning of parameters:

- we can first set values: R_2, R_4, C_1, C_2 to obtain the desired value of ω_0,
- then by adjusting R_1 we only change Q - without H and ω_0 changes,
- finally we set R_3 which changes only H - without changes in Q and ω_0.

OTA-C implementation



(a)



(b)

Fig. 40. Product realization of second-order lowpass section (a) and its normalized OTA-C implementation (b).

Mixed mode voltage / current OTA-C implementation

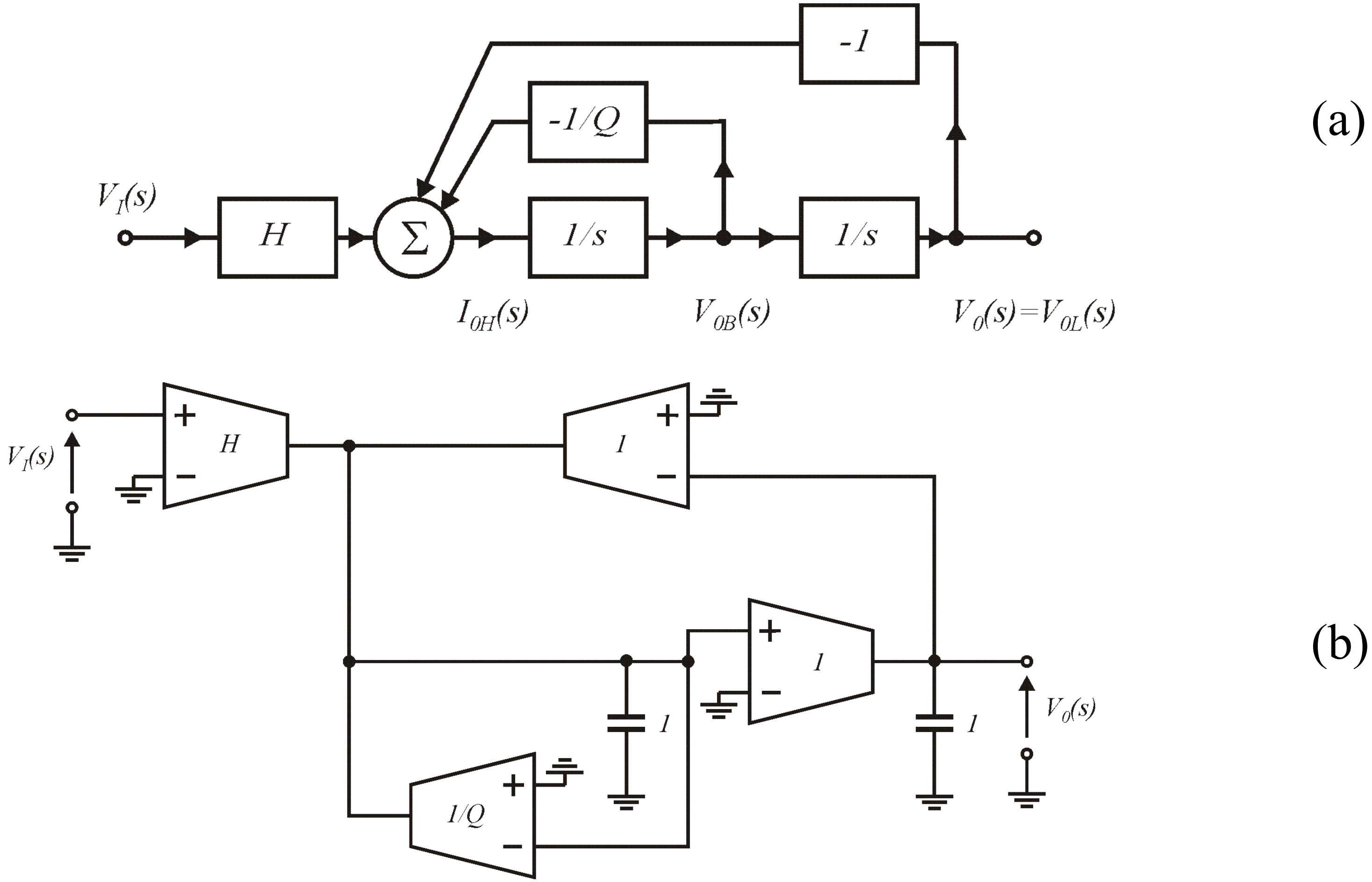


Fig. 41. Product realization of second-order lowpass section (a) and its normalized, mixed mode voltage and current OTA-C implementation (b).

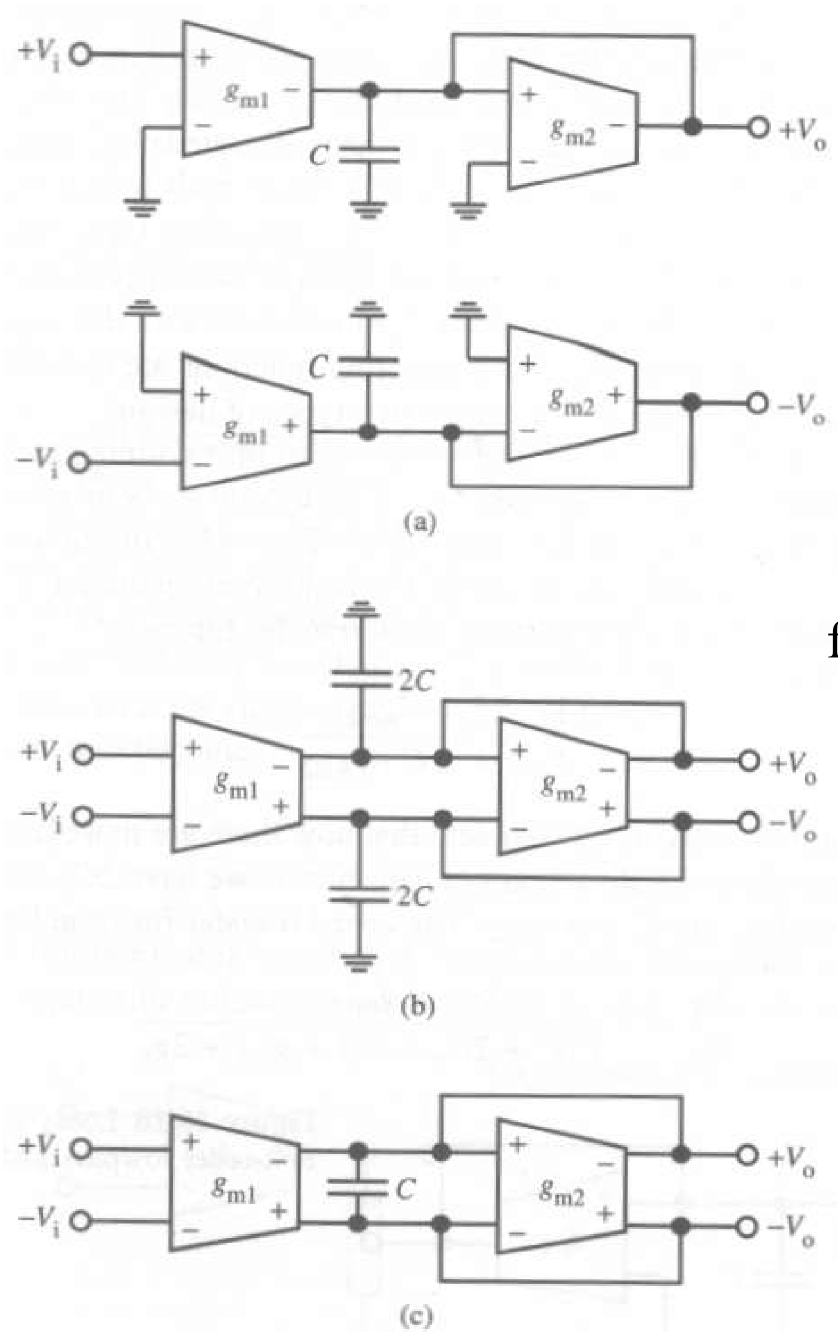
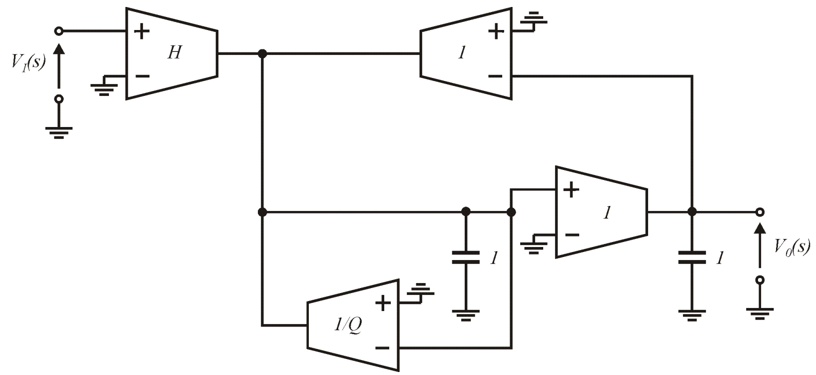


Fig. 42. Conversion of the single output to fully differential OTA-C implementation [1].

Fully differential OTA-C implementation



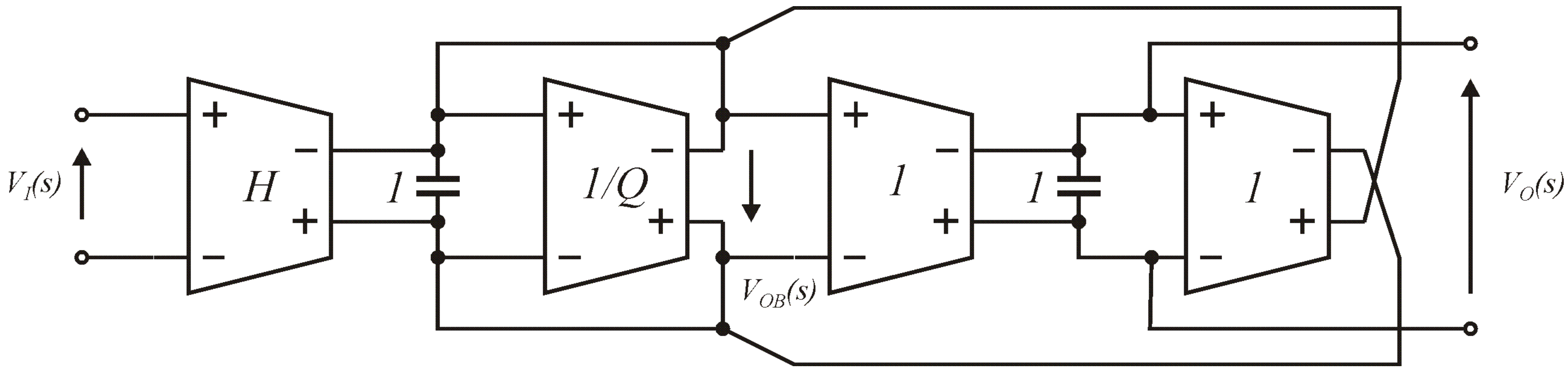


Fig. 43. Fully differential version of the filter from Fig.41.

Calculation example no 3

Please design fully differential lowpass 2-order OTA-C filter with quality factor Q=4, gain H=1 and natural frequency f_0=50MHz by the use of:

a) denormalization and

b) transfer function comparison.

OTAs with gm values in the range of 10-100µS are available.

A) – solution with the use of denormalization. Denormalizing radial frequency:

$$\omega_0 = 2\pi f_0 = 2\pi 50MHz = 314{,}16Mrad/\sec$$

Since the available transconductances are in the range of 10-100μS, we choose the main transconductance equal to 100μS and hence the value of the normalizing resistance is equal to:

$$R_n = \frac{1}{R_S}R \qquad G_n = R_S G \qquad R_S = G_n/G \qquad G = G_n/R_S$$

$$R_S = 1/100\mu S = 10k\Omega$$

while capacitor values are equal to:

$$C = C_n/\left(R_S\omega_S\right) = 1/(10k\Omega 314{,}16Mrad/\sec) = 0{,}31831pF$$

Transconductance of a $1/Q$ normalized amplifier reach the value of:

$$G_{1/Q} = G_n / R_S = 1/4/10k\Omega = 25\mu S$$

Thus the final filter scheme will be as follows:

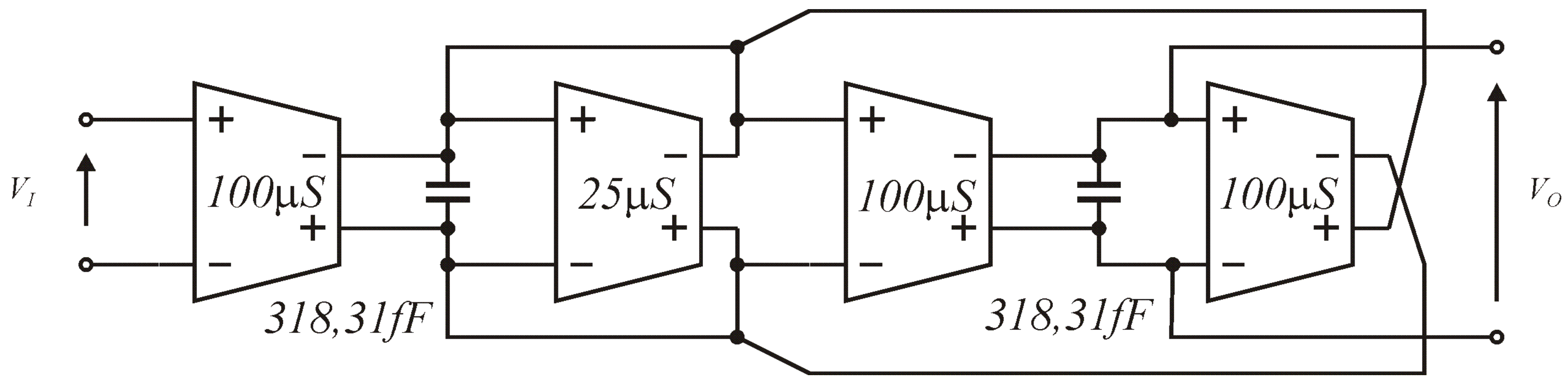


Fig. 44. Calculated values of filter devices using the denormalization method.

B) Transfer function comparison method.

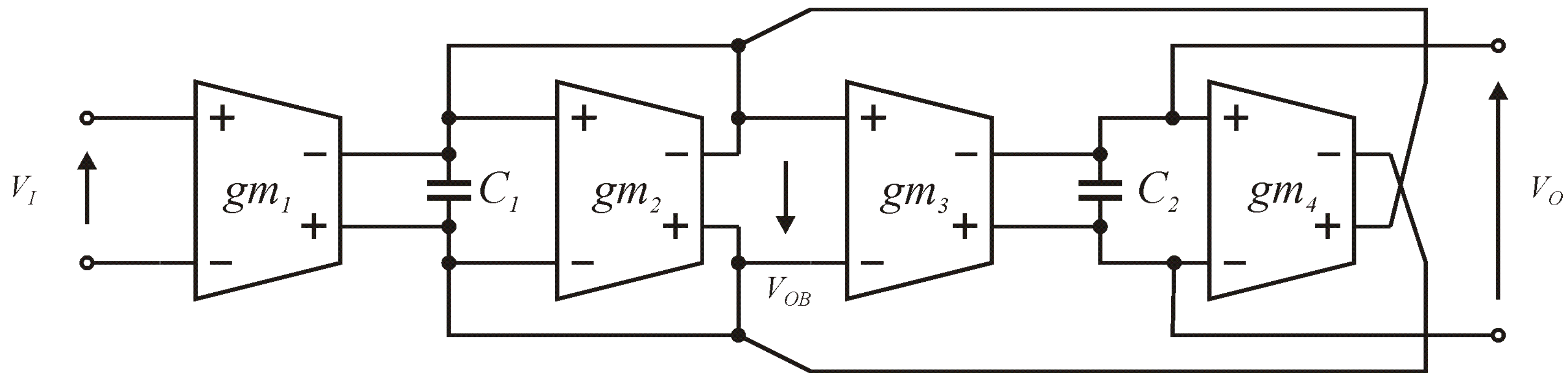


Fig. 45. General schematic of lowpass second order OTA-C section.

$$Y = sC_1 + gm_2 \qquad V_0(s) = V_{OB}(s)\frac{gm_3}{sC_2}$$

$$V_{OB}(s) = V_I(s)\frac{gm_1}{Y} - V_0(s)\frac{gm_4}{Y}$$

B) Transfer function comparison method – cont. The determination of the values of the output voltage and intermediate voltage from the system gives the following results:

$$\frac{V_{OB}(s)}{V_I(s)} = \frac{s\dfrac{gm_1 gm_3{}^2}{C_1}}{s^2 + \dfrac{gm_2}{C_1}s + \dfrac{gm_3 gm_4}{C_1 C_2}}$$

$$\frac{V_0(s)}{V_I(s)} = \frac{\dfrac{gm_1 gm_3}{C_1 C_2}}{s^2 + \dfrac{gm_2}{C_1}s + \dfrac{gm_3 gm_4}{C_1 C_2}}$$

Comparing the above expression with the general LP II order section transfer function one can obtain:

$$T(s) = H\frac{\omega_0^2}{s^2 + \dfrac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0^2 = \frac{gm_3 gm_4}{C_1 C_2} \quad => \quad \omega_0 = \sqrt{\frac{gm_3 gm_4}{C_1 C_2}}$$

$$H\omega_0^2 = \frac{gm_1 gm_3}{C_1 C_2} \quad => \quad H = \frac{gm_1}{gm_4}$$

$$\frac{\omega_0}{Q} = \frac{gm_2}{C_1} \quad => \quad Q = \sqrt{\frac{C_1}{C_2}\frac{gm_3 gm_4}{gm_2{}^2}}$$

B) Transfer function comparison method – cont. We have a larger freedom for device's values selection than for denormalization of the prototype method. For example, we can choose the same values of transconductances, each equal to 100μS and then capacitors determine from the formulas: $gm_1 = gm_2 = gm_3 = gm_4 = 100\mu S$

$$Q = 4 = \sqrt{\frac{C_1}{C_2}\frac{gm_3 gm_4}{gm_2{}^2}} = \sqrt{\frac{C_1}{C_2}} \quad => \quad C_1 = 16C_2$$

$$\omega_0 = 2\pi f_0 = 314{,}16M[rad/\sec] = \sqrt{\frac{gm_3 gm_4}{C_1 C_2}} = \sqrt{\frac{100uS100uS}{16C_2C_2}} = \frac{100uS}{4C_2} \quad =>$$

$$C_2 = \frac{100uS}{4 \bullet 314{,}16Mrad/\sec} = 79{,}57\,fF \qquad C_1 = 16C_2 = 1273{,}12\,fF$$

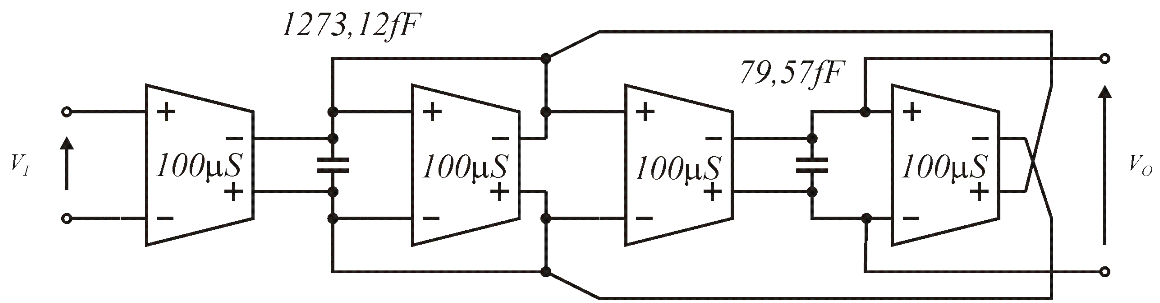
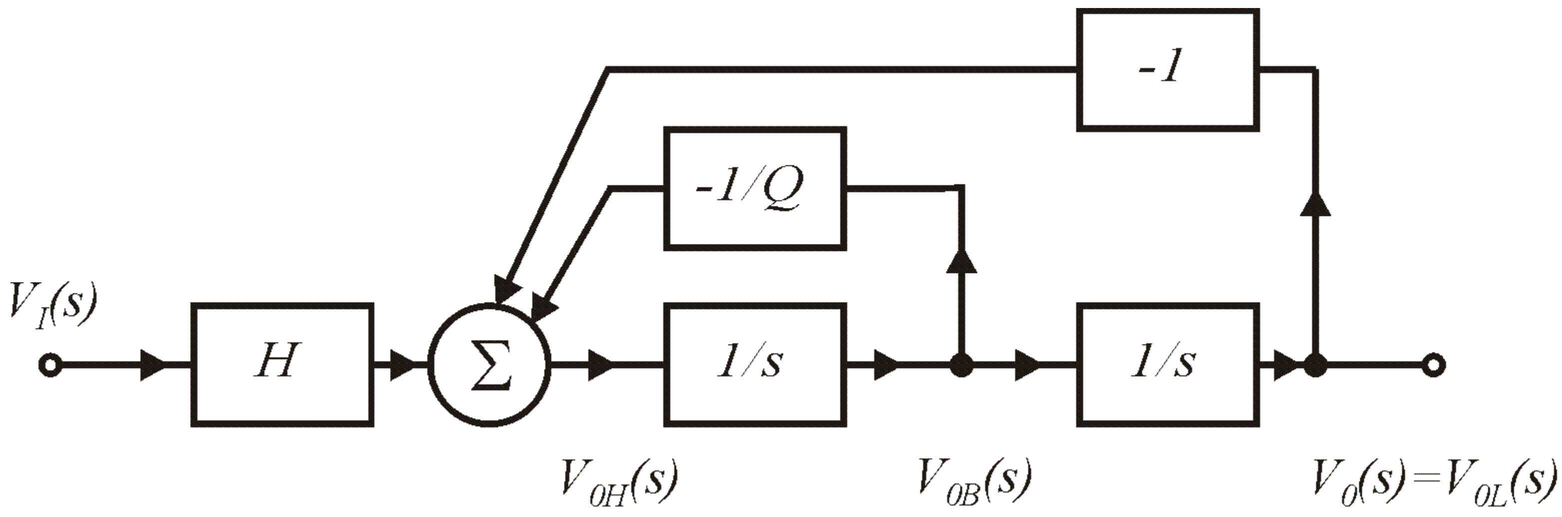


Fig. 46. Calculated values of filter devices using the transfer function comparison method.

Homework no 2

Please design a second order lowpass filter with quality factor Q=5, gain H=2 and natural frequency f_0=5kHz using second generation current conveyors. Use the product realisation of biquad section as shown in the figure below for the design. Values of devices have to be calculated using denormalization as well as transfer function comparison methods.



II – order sections, all types of transfer functions [1]

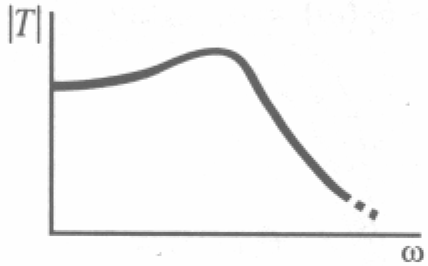
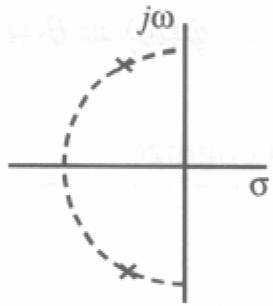
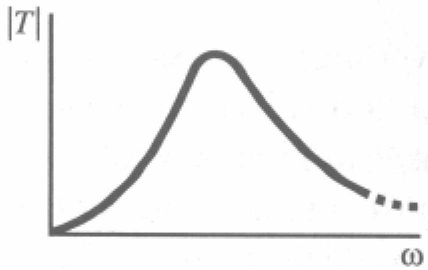
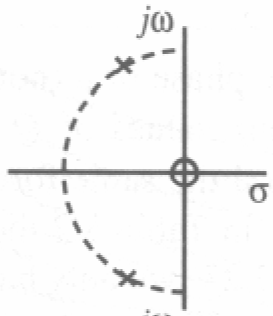
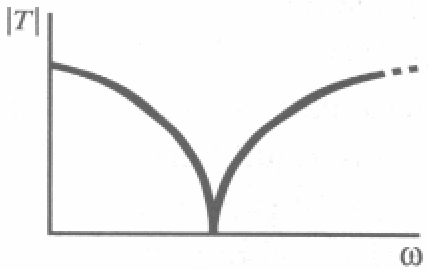
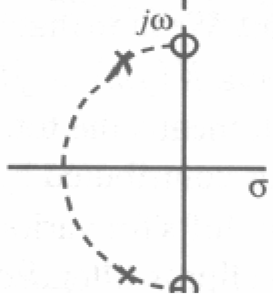
<table>
<tr><th></th><th>Frequency Response</th><th>Poles/Zeros</th><th>Name</th></tr>
<tr><td>$$T_{LP} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$</td><td></td><td></td><td>Lowpass</td></tr>
<tr><td>$$T_{BP} = \frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$</td><td></td><td></td><td>Bandpass</td></tr>
<tr><td>$$T_{BE} = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$</td><td></td><td></td><td>Bandstop "notch"</td></tr>
</table>

Fig. 47a. II-order sections - magnitude responses and poles and zeros locations [1].

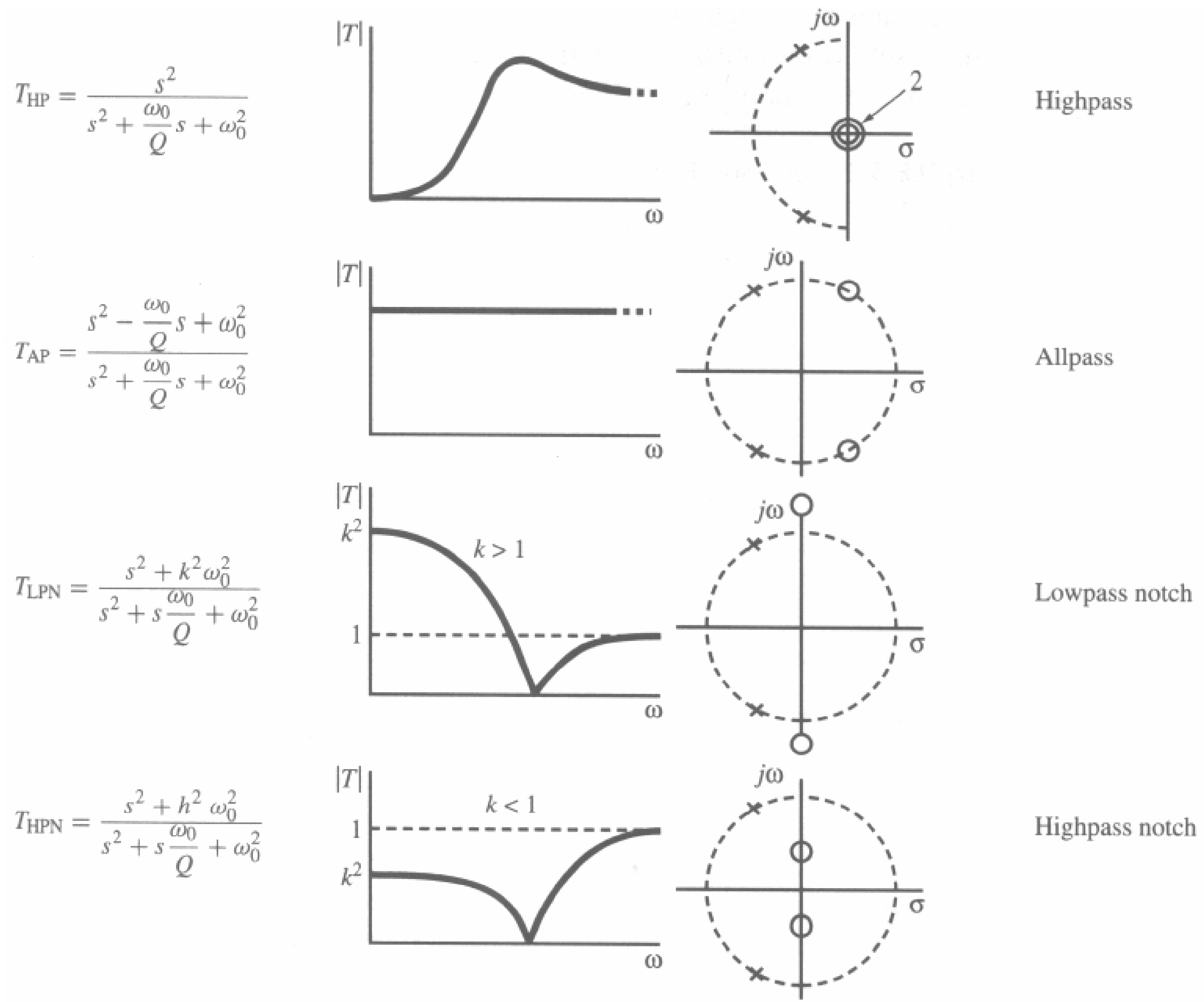


Fig. 47b. II-order sections - magnitude responses and poles and zeros locations [1].

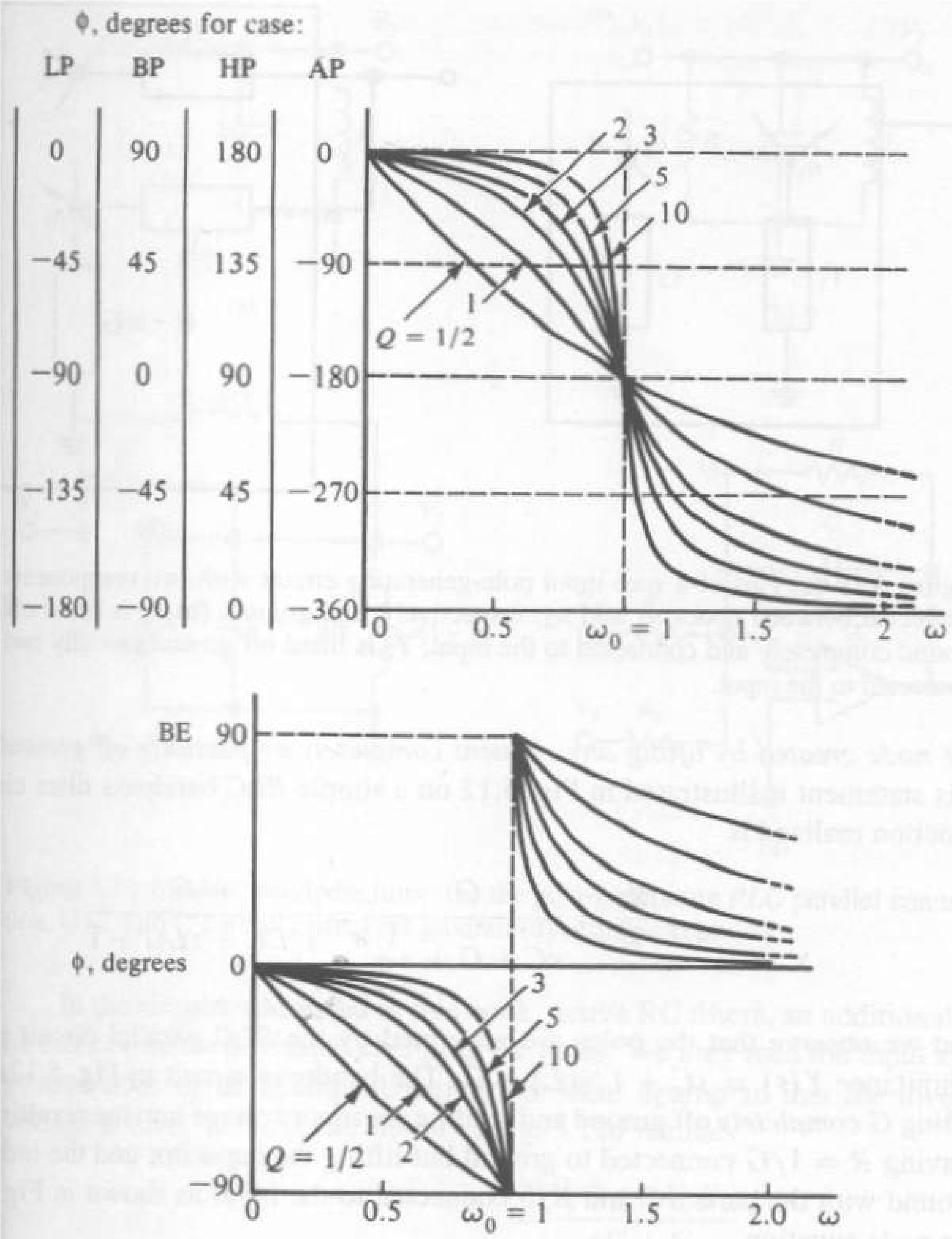


Fig. 48. II-order sections - phase responses [1].

Higher order filters

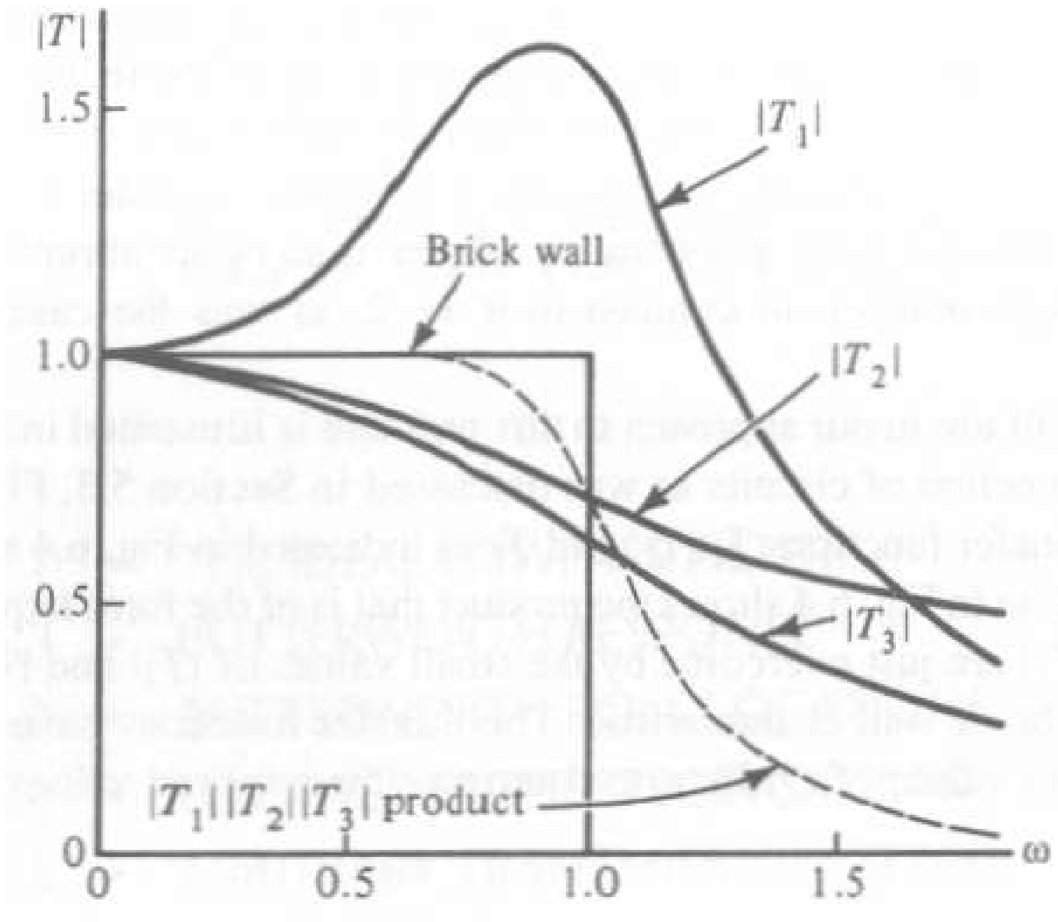


Fig. 49. II-order sections in cascade configuration can gain in more ideal (brick wall) lowpass frequency response [1].

Popular approximations

- Butterworth, maximally flat magnitude
- Chebyshev, equal-ripple magnitude
- Inverse Chebyshev
- Cauer
- Elliptic
- Bessel-Thomson, and others

Butterworth response [1]

Transfer function *T(jω)* can be written as the sum of real and imaginary parts:

$$T(j\omega) = \mathrm{Re}\,T(j\omega) + j\,\mathrm{Im}\,T(j\omega)$$

The real part is symmetrical in respect to the Y axis and the imaginary part is symmetrical in respect to the origin of the coordinate system:

$$T(-j\omega) = \mathrm{Re}\,T(j\omega) - j\,\mathrm{Im}\,T(j\omega)$$

T(-jω) is conjugate of *T(jω):*

$$T(-j\omega) = T^*(j\omega)$$

Because:

$$T(j\omega)\cdot T^*(j\omega) = \left(\mathrm{Re}\,T\right)^2 + \left(\mathrm{Im}\,T\right)^2 = \left|T(j\omega)\right|^2$$

So:

$$\left|T(j\omega)\right|^2 = T(j\omega)\cdot T(-j\omega)$$

Butterworth response – cont. [1]

We can express the square of the transfer function module as a fraction of two polynomials, where *n* expresses the degree of a polynomial and the polynomials *A* and *B* must be even:

$$\left|T_n(j\omega)\right|^2 = \frac{\left|N_n(j\omega)\right|^2}{\left|D_n(j\omega)\right|^2} = \frac{A(\omega^2)}{B(\omega^2)}$$

Let define a characteristic function *K(s)* being the deviation of the inverse of the transfer function module from unity:

$$\left|K(j\omega)\right|^2 = \left|T(j\omega)\right|^{-2} - 1 = \frac{B(\omega^2) - A(\omega^2)}{A(\omega^2)}$$

The value of the characteristic function module in the filter band should be equal to zero and infinity beyond this band.

Butterworth response – cont. [1]

An exemplary graph of the transfer function module is shown in Fig. 50 (a) and the characteristic function module in Fig. 50 (b).

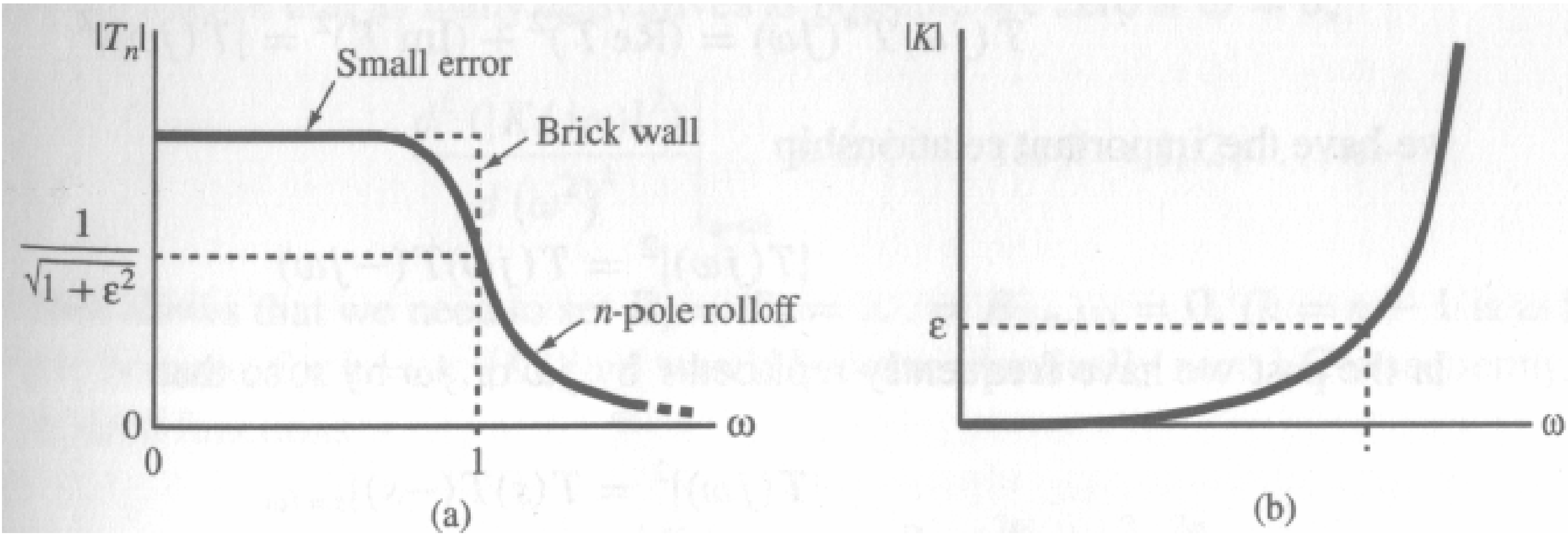


Fig. 50. Plots of the transfer function module (a) and the characteristic function module (b) for the low-pass filter [1].

Butterworth response – cont. [1]

For a lowpass filter, the normalized value of polynomial A is equal to 1. Thus, the square of the transfer function module can be described as:

$$\left|T_n(j\omega)\right|^2 = \frac{\left|N_n(j\omega)\right|^2}{\left|D_n(j\omega)\right|^2} = \frac{1}{B(\omega^2)} = \frac{1}{1 + B_2\omega^2 + B_4\omega^4 + B_6\omega^6 + \ldots + B_{2n}\omega^{2n}}$$

Hence, the square value of the characteristic function module will be equal to:

$$\left|K_n(j\omega)\right|^2 = B_2\omega^2 + B_4\omega^4 + B_6\omega^6 + \ldots + B_{2n}\omega^{2n}$$

Now assume that the characteristic function |K(jω)| should be as flat as possible - so its subsequent derivatives for ω=0 should be equal to zero, hence:

$$\left.\frac{d^k\left(\left|K(j\omega)\right|^2\right)}{d\left(\omega^2\right)^k}\right|_{\omega=0} = 0 \quad \text{for } k = 1,2,\ldots,n-1$$

Butterworth response – cont. [1]

From the previous expression, it follows that $B_2=B_4=B_{2(n-1)}=0$ and the characteristic function takes the form:

$$\left|K_n(j\omega)\right|^2 = B_{2n}\omega^{2n} = \varepsilon^2\omega^{2n}$$

The square of the transfer function module is equal (for designation $B_{2n}=\varepsilon^2$):

$$\left|T_n(j\omega)\right|^2 = \frac{1}{1+B_{2n}\omega^{2n}} = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

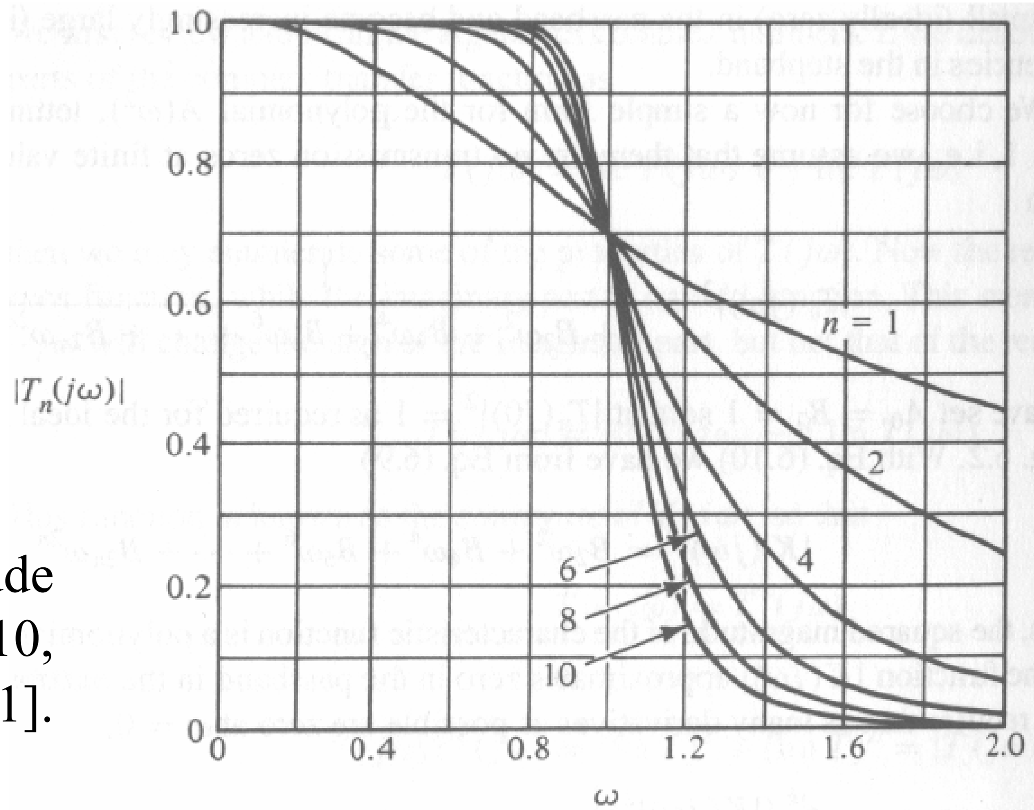


Fig. 51. Butterworth amplitude responses for orders from n=1 do 10, ε=1 [1].

Butterworth response – cont. [1]

If we assume that $\varepsilon=1$, then:

$$\left|T_n(j\omega)\right|^2 = \frac{1}{1+\omega^{2n}}$$

The properties resulting from the above equation are as follows:

• there is no zeros of transfer function,

• $|T_n(j0)|=1$ regardless of n,

• $|T_n(j1)|=1/2^{0,5}=0,707$ which corresponds to -3dB on the amplitude characteristic,

• for large ω we have a decrease in amplitude responses equal to $20n$/decade.

Butterworth response – cont. [1]

It can be mathematically proved that the positions of the Butterworth transfer function poles are uniform on the unit circle. This is shown in Fig. 52.

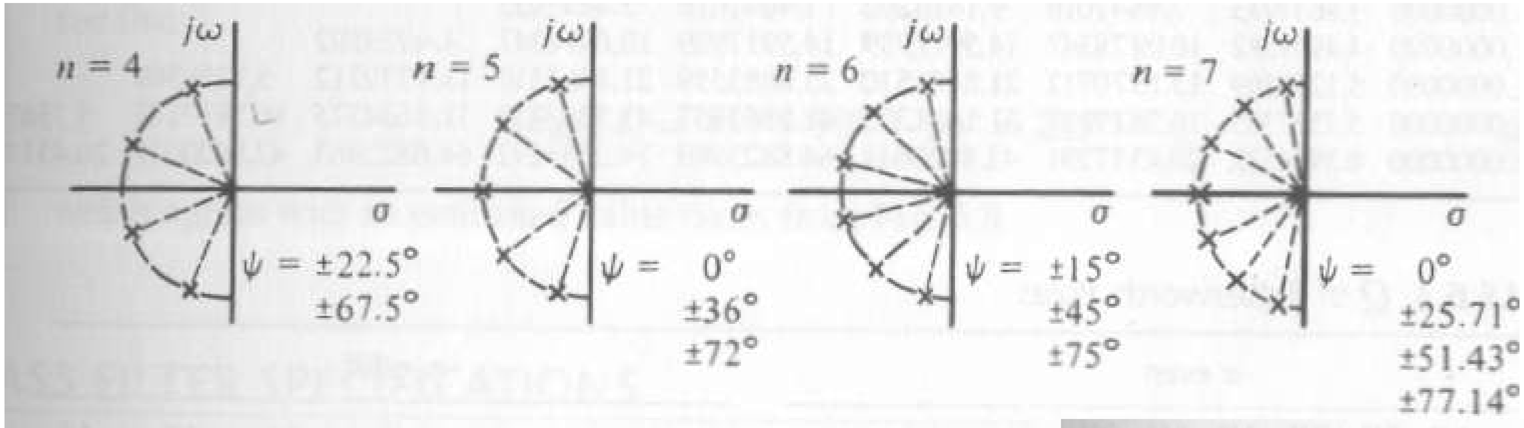


Fig. 52. Location of Butterworth transfer function poles for orders from n = 1 to 7 [1].

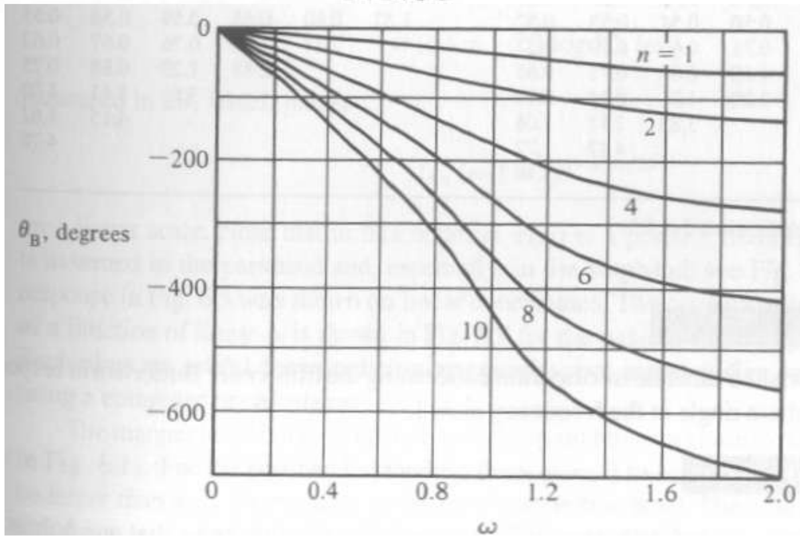


Fig. 53. Butterworth's phase curves for orders from n = 1 to 10 [1].

Butterworth response – cont. [1]

Locations of the Butterworth transfer function poles [1]:

$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
−0.7071068	−0.5000000	−0.3826834	−0.8090170	−0.2588190	−0.9009689	−0.1950903	−0.9396926	−0.1564345
$\pm j$0.7071068	$\pm j$0.8660254	$\pm j$0.9238795	$\pm j$0.5877852	$\pm j$0.9659258	$\pm j$0.4338837	$\pm j$0.9807853	$\pm j$0.3420201	$\pm j$0.9876883
	−1.0000000	−0.9238795	−0.3090170	−0.7071068	−0.2225209	0.5555702	−0.1736482	−0.4539905
		$\pm j$0.3826834	$\pm j$0.9510565	$\pm j$0.7071068	$\pm j$0.9649279	$\pm j$0.8314696	$\pm j$0.9848078	$\pm j$0.8910065
			−1.0000000	−0.9659258	0.6234898	−0.8314696	−0.5000000	−0.7071068
				$\pm j$0.2588190	$\pm j$0.7818315	$\pm j$0.5555702	$\pm j$0.8660254	$\pm j$0.7071068
					−1.0000000	−0.9807853	−0.7660444	−0.8910065
						$\pm j$0.1950903	$\pm j$0.6427876	$\pm j$0.4539905
							−1.0000000	−0.9876883
								$\pm j$0.1564345

Coefficients of the denominator polynomial [1]: $B_n(s) = s^n + \sum_{i=1}^{n-1} a_i s^i$

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
2	1.0000000	1.4142136								
3	1.0000000	2.0000000	2.0000000							
4	1.0000000	2.6131259	3.4142136	2.6131259						
5	1.0000000	3.2360680	5.2360680	5.2360680	3.2360680					
6	1.0000000	3.8637033	7.4641016	9.1416202	7.4641016	3.8637033				
7	1.0000000	4.4939592	10.0978347	14.5917939	14.5917939	10.0978347	4.4939592			
8	1.0000000	5.1258309	13.1370712	21.8461510	25.6883559	21.8461510	13.1370712	5.1258309		
9	1.0000000	5.7587705	16.5817187	31.1634375	41.9863857	41.9863857	31.1634375	16.5817187	5.7587705	
10	1.0000000	6.3924532	20.4317291	42.8020611	64.8823963	74.2334292	64.8823963	42.8020611	20.4317291	6.3924532

Butterworth response – cont. [1]

Quality factors Q of Butterworth's poles: [1]

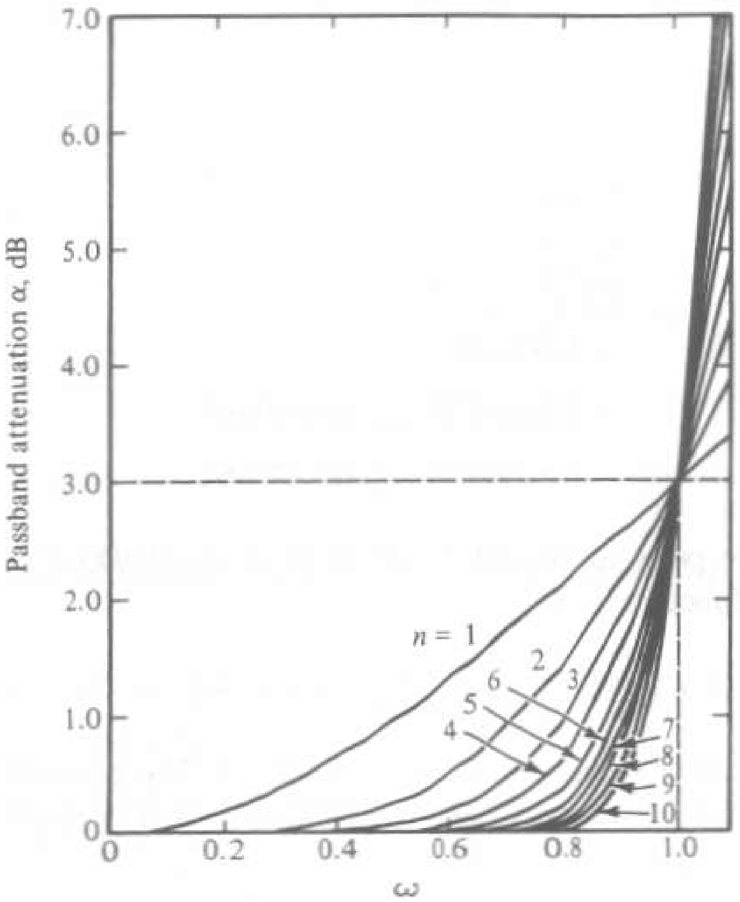
n even								n odd[a]						
2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**3**	**5**	**7**	**9**	**11**	**13**	**15**
0.71	0.54	0.52	0.51	0.51	0.50	0.50	0.50	1.00	0.62	0.55	0.53	0.52	0.51	0.51
	1.31	0.71	0.60	0.56	0.54	0.53	0.52		1.62	0.80	0.65	0.59	0.56	0.55
		1.93	0.90	0.71	0.63	0.59	0.57			2.24	1.00	0.76	0.67	0.62
			2.56	1.10	0.82	0.71	0.65				2.88	1.20	0.88	0.75
				3.20	1.31	0.94	0.79					3.51	1.41	1.00
					3.83	1.51	1.06						4.15	1.62
						4.47	1.72							4.78
							5.10							

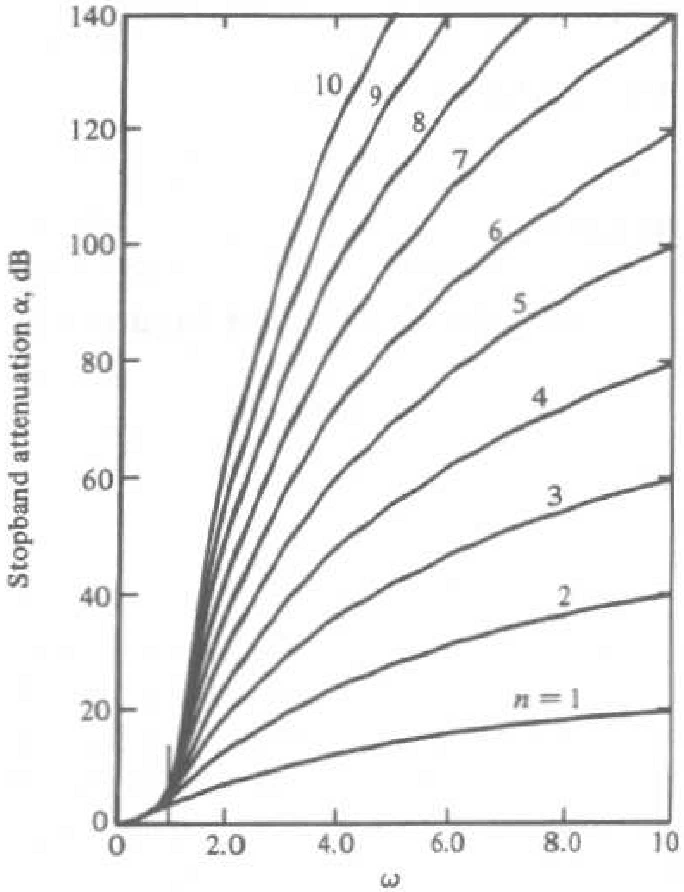
[a] For n odd there is also a real pole for which $Q = 0.5$.

Butterworth response – cont. [1]

In order to determine which order of the filter is sufficient, one can use the following formula or use the monograms available in the literature [1].

$$\left|T_n(j\omega)\right|^2 = \frac{1}{1+\omega^{2n}}$$





Calculation example no 4 – cascade realization of the filter

A lowpass filter of 7-order using Butterworth approximation should be designed. The 3dB passband of the filter should be equal to 50kHz and cascade implementation should be used.

Solution: From the table of quality factorses of Butterworth transfer function, we read the following values: Q=0.55, Q=0.8 and Q=2.24 and the need for real pole. Hence our filter will consist of 3 biquad sections (implemented using any amplifier, e.g. OTA, OA, CCII+) and a section with a real pole (e.g. RC).

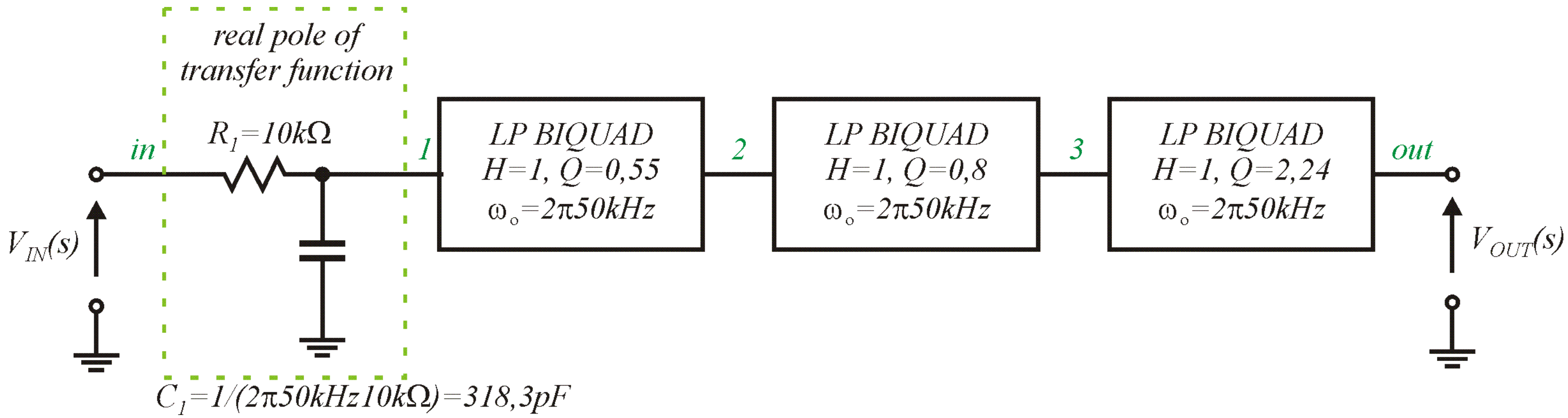


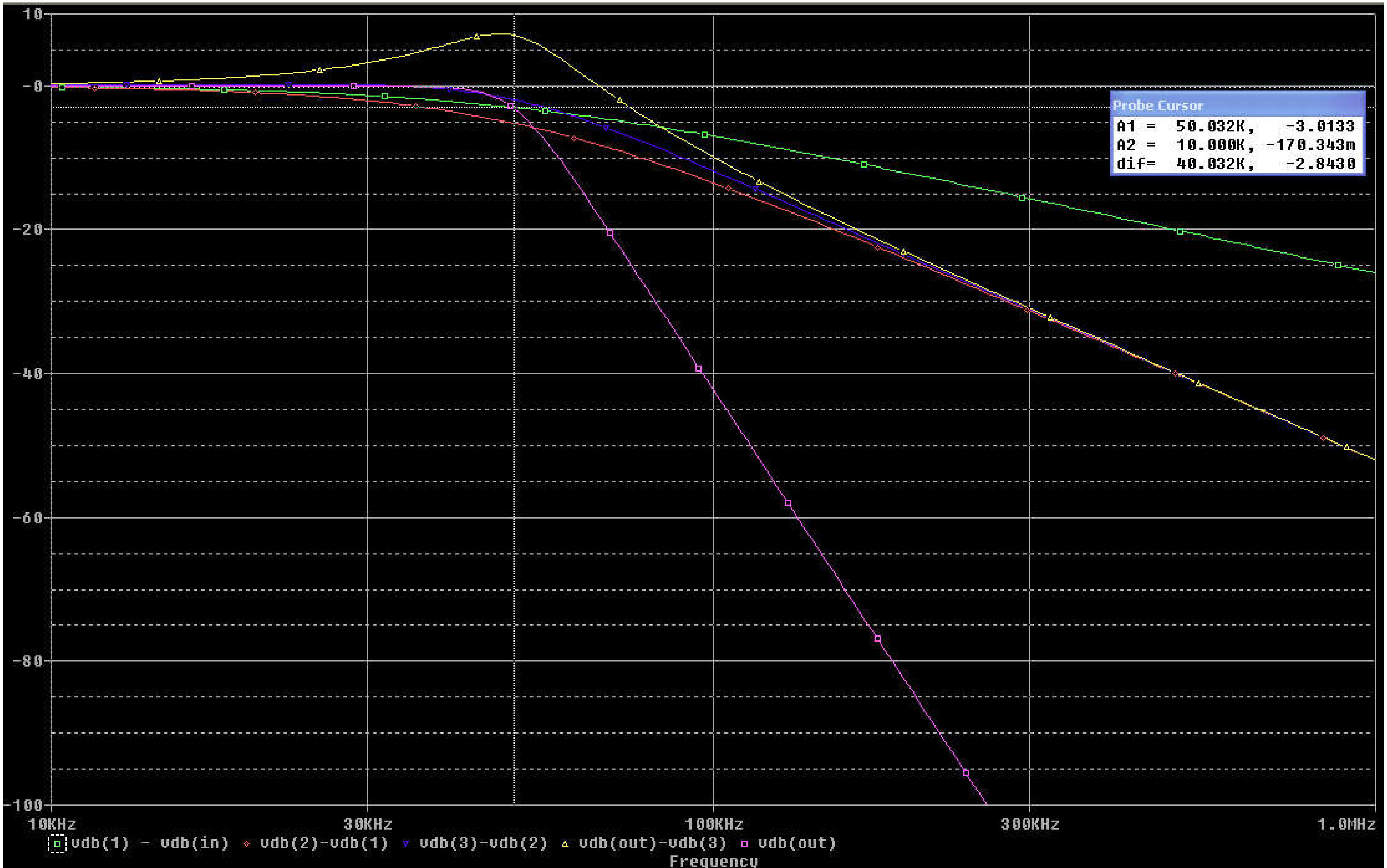
Fig. 54. Implementation of the filter from the calculation example.

Calculation example – results checking using the PSPICE simulator

PSPICE file:

```
VII - order, lowpass, Butterworth's filter
.param f=50k
.param w={2*3.1415*f}
Vin in 0 dc 0 ac 1 sin(0 1 1)                         -- input
.subckt biquad_lp_id out in params: wo=1 Q=1         -- declaration of biquad sec.
E1 out 0 laplace {V(in)}={wo*wo/(s*s+wo/Q*s+wo*wo)}
.ends
.subckt real_pole out in params: wo=1                 -- decl. of real pole section
E1 out 0 laplace {V(in)}={wo/(s+wo)}
.ends
************** Filter sections
X1 2 1 biquad_lp_id params: wo={w} Q=0.55
X2 3 2  biquad_lp_id params: wo={w} Q=0.8
X3 out 3  biquad_lp_id params: wo={w} Q=2.24
X4 1 in real_pole   params: wo={w}
RL out 0 1                                            -- load
************** Analyzes
.ac dec 100 .001 100
.tran 0.01 10 9 0.01
.probe
.end
```

PSPICE simulation results



RLC prototype based filters [1]

Although RLC filters are not active integrated filters they are very useful in the design of active filters due to:

• low sensitivity of filter parameters in respect to filter components parameters,

• those filters are widely used today for high frequencies or for powerless systems,

• active filters are designed based on passive prototypes what gives the transfer of good properties to their active counterparts,

• already known knowledge of approximation and synthesis of filters can be used.

RLC filters synthesis [1]

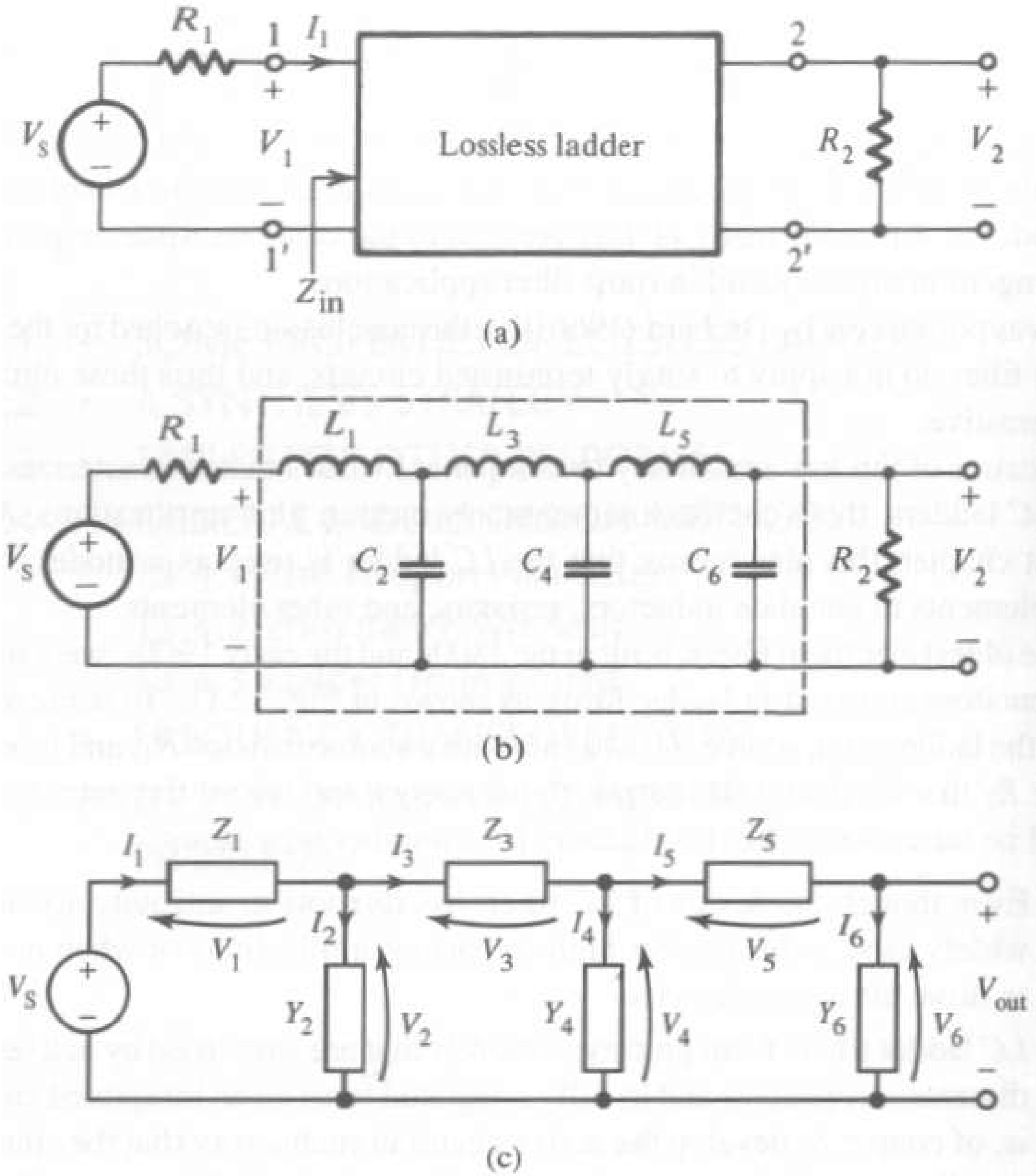


Fig. 55. Double terminated RLC filter: (a) general representation, (b) VI order lowpass filter, (c) impedance and admittance representation of the filter [1].

RLC filters synthesis [1]-cont.

Input impedance of the filter in Fig. 55(a) is equal to:

$$Z_{in} = R_{in} + jX_{in}$$

Input current can be calculated as:

$$I_1 = \frac{V_S}{R_1 + Z_{in}}$$

Since the system is lossless, all power entering the filter is lost on the load resistance:

$$P_1 = R_{in} \left| I_1(j\omega) \right|^2 = \frac{\left| V_2(j\omega) \right|^2}{R_2}$$ thus:

$$\frac{R_{in} \left| V_S(j\omega) \right|^2}{\left| R_1 + Z_{in} \right|^2} = \frac{\left| V_2(j\omega) \right|^2}{R_2}$$ what gives: $$\left| \frac{V_2(j\omega)}{V_S(j\omega)} \right|^2 = \left| T(j\omega) \right|^2 = \frac{R_2 R_{in}}{\left| R_1 + Z_{in} \right|^2}$$

RLC filters synthesis [1]-cont.

The transfer function *T(s)* in now multiplied by the coefficient *sqrt(4R1/R2)* to obtain the *H(S)* normalized to 1 for the pass band:

$$\left|H(j\omega)\right|^2 = \frac{4R_1}{R_2}\left|\frac{V_2(j\omega)}{V_S(j\omega)}\right|^2 = \frac{4R_1 R_{in}}{\left|R_1 + Z_{in}\right|^2} \leq 1$$

Using previous equations one can obtain

$$\left|H(j\omega)\right|^2 = 1 - \frac{\left|R_1 - Z_{in}\right|^2}{\left|R_1 + Z_{in}\right|^2} = 1 - \left|\rho(j\omega)\right|^2$$

The $\rho(j\omega)$ term is cold reflection coefficient. Using above equations it can be find as:

$$\left|\rho(j\omega)\right|^2 = \left|\rho(s)\right|\left|\rho(-s)\right|_{s=j\omega} = \frac{\left|R_1 - Z_{in}(j\omega)\right|^2}{\left|R_1 + Z_{in}(j\omega)\right|^2}$$

RLC filters synthesis [1]-cont.

And hence: $$\left|\rho(j\omega)\right|^2 = \left|\rho(s)\right|\left|\rho(-s)\right|_{s=j\omega} = \pm\frac{R_1 - Z_{in}(s)}{R_1 + Z_{in}(s)}$$

What gives 2 possible input impedances:

$$Z_{in}(s) = R_1 \frac{1+\rho(s)}{1-\rho(s)} \quad \text{lub} \quad Z_{in}(s) = R_1 \frac{1-\rho(s)}{1+\rho(s)}$$

The above two impedances implement our desired transfer function. The procedure for synthesizing the RLC prototype is thus as follow:

- we have a given *H(s)* and from here we find $\rho(s)$,
- then we set values of R_1 and R_2 and get two possible values of $Z_{in}(s)$ realizing the given transfer function,
- finally we can arrange the RLC ladder with the designated impedance $Z_{in}(s)$.

RLC filters synthesis [1]- cont.

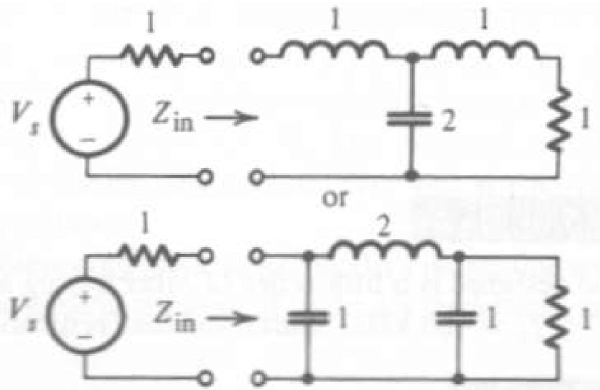
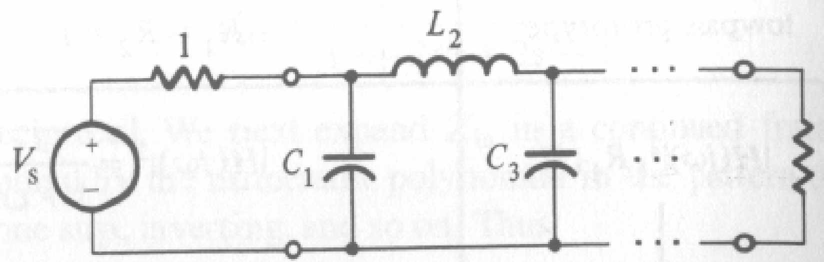
Procedure to obtain doubly terminated lowpass prototype	Example: Butterworth $n = 3$, $R_1 = R_2 = 1$
$\lvert H(j\omega)\rvert^2, R_1, R_2$ ↓	$\lvert H(j\omega)\rvert^2 = \dfrac{1}{1+\omega^6}$
$\lvert \rho(j\omega)\rvert^2$ ↓	$\lvert \rho(j\omega)\rvert^2 = \dfrac{\omega^6}{1+\omega^6}$
$\rho(s)$ ↓	$\rho(s) = \dfrac{\pm s^3}{s^3 + 2s^2 + 2s + 1}$
$Z_{in}(s)$ ↓	$Z_{in} = \left(\dfrac{2s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1}\right)^{\pm 1}$
Continued fraction expansion ↓	$\left.\begin{matrix} Z_{in} \\ \text{and} \\ 1/Z_{in} \end{matrix}\right\} = s + \cfrac{1}{2s + \cfrac{1}{s + \cfrac{1}{1}}}$
Two circuit realizations	

Fig. 56. RLC filters synthesis procedure example [1].

RLC filters synthesis [1]-cont.



n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9	L_{10}
2	1.414	1.414								
3	1.000	2.000	1.000							
4	0.7654	1.848	1.848	0.7654						
5	0.6180	1.618	2.000	1.618	0.6180					
6	0.5176	1.414	1.932	1.932	1.414	0.5176				
7	0.4450	1.247	1.802	2.000	1.802	1.247	0.4450			
8	0.3902	1.111	1.663	1.962	1.962	1.663	1.111	0.3902		
9	0.3473	1.000	1.532	1.879	2.000	1.879	1.532	1.000	0.3473	
10	0.3129	0.9080	1.414	1.782	1.975	1.975	1.782	1.414	0.9080	0.3129
n	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}

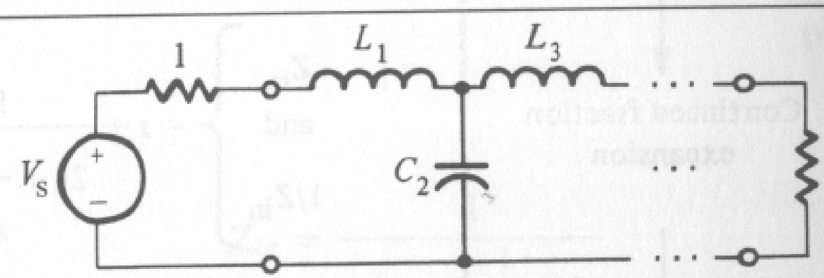


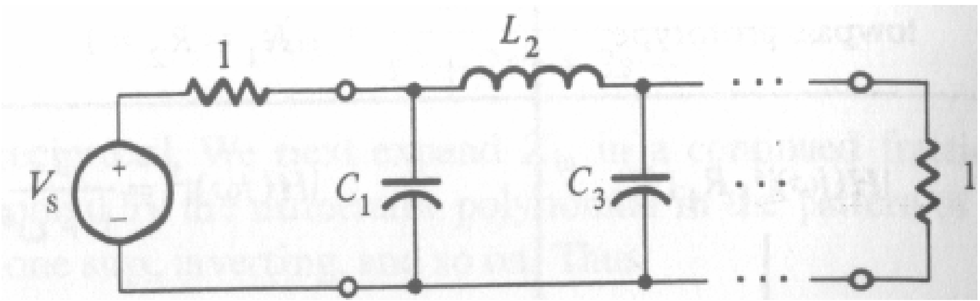
Fig. 57. Values of normalised Butterworth RLC filter prototype [1].

Calculation example – VII order RLC Butterworth filter (#5)

A low-pass Butterworth, RLC, VII order filter with 3dB frequency of 50kHz should be designed

Solution: choose one of the implementations from the table, select a normalizing resistance equal to, for example, 20kΩ and then based on the normalised values table (Fig. 57) calculate the actual values as:

$$C = \frac{1}{R_S \omega_S} C_n \qquad L = \frac{R_S}{\omega_S} L_n \qquad R = R_S R_n \qquad \omega_S = 2\pi 50kHz$$



Calculation results:

R_1=R_2=20kΩ, C_1=70,82pF, L_2=79,38mH, C_3= 286,8pF, L_4=127,3mH, C_5=286,8pF, L_6= 79,38mH, C_7= 70,82pF

Calculation example (#5) PSPICE silulation results

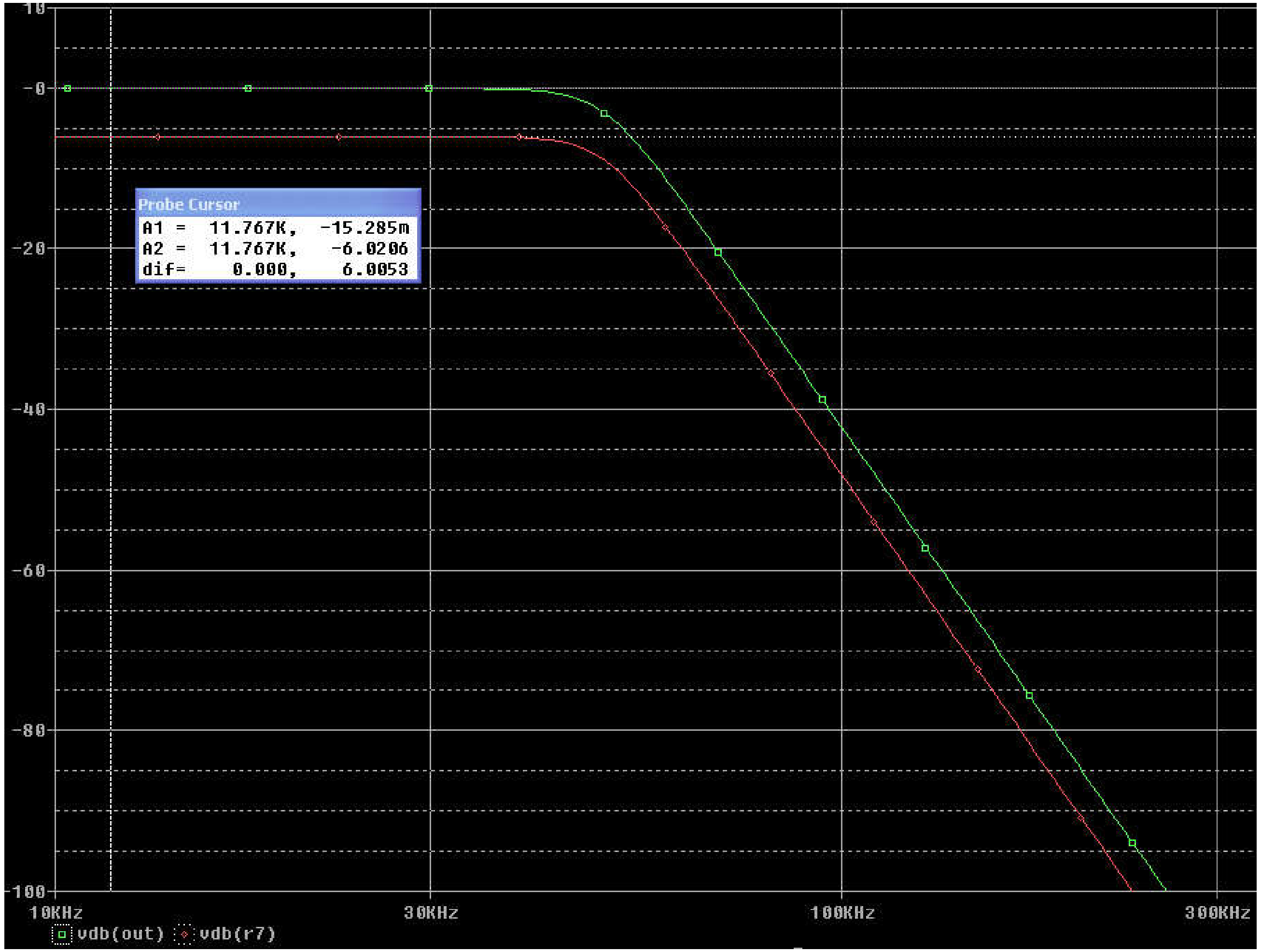


Fig. 58. Results of the PSPICE simulation of the Butterworth filter realized by the cascade technique (green) and using the RLC prototype (red).

Homework no 3

Please design a 6th order filter using Butterworth approximation of transfer function. Please use cascade implementation (with OTA-C biquads) and also RLC ladder implementation. Filter should exhibit 3dB passband for frequency equal to 100kHz. Available are OTA amplifiers of 100μS transconductance value, resistors of values in the range of 10 - 30kΩ and capacitors of any value. In order to verify design correctness the PSPICE simulation of the filters should be also performed.

Active filters based on RLC ladder filters

Due to the good sensitivity properties, RLC filters are often a pattern that is then used for implementation one of following ways:

- direct simulation of the RLC prototype,

  - direct devices replacement method (+ impedance transformations),

  - using Bruton's transformation,

  - using Gorski – Popiel's technique,

- simulation of the signal flow graph.

Direct simulation of RLC prototype – gyrator

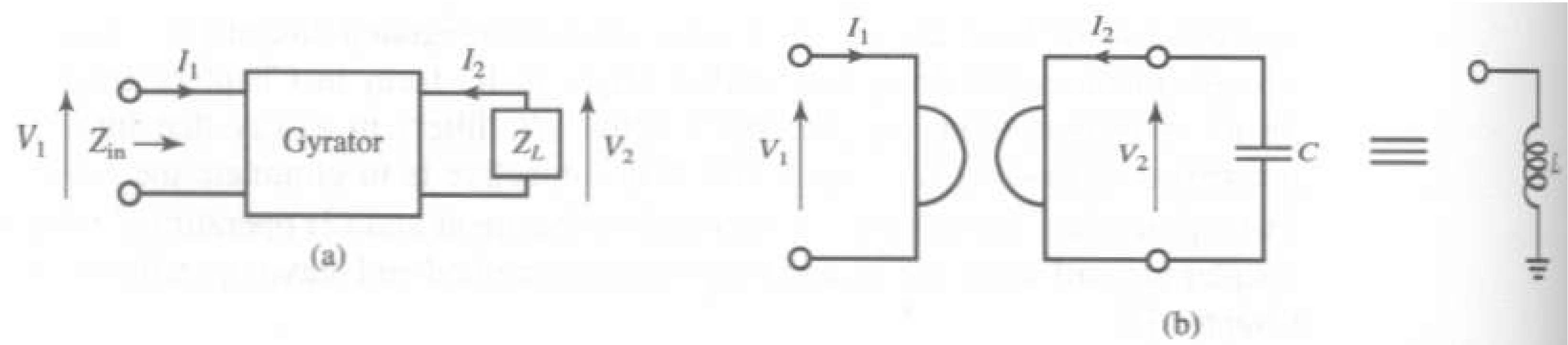


Fig. 58. Gyrator (a) and its symbol (b) and use as a coil imitation [1].

The gyrator is described by the following equations, where *gm* is a gyration conductance parameter:

$$I_1 = -gmV_2 \text{ oraz } I_2 = gmV_1$$

If the impedance Z_L is placed on terminals ”2” of the gyrator, impedance seen from ”1” terminals will be equal to:

$$Z_{in}(s) = \frac{1}{gm^2}\left(-\frac{I_2}{V_2}\right) = \frac{1}{gm^2}\frac{1}{Z_L(s)}$$

Direct simulation of RLC prototype – gyrator, cont.

So when the gyrator is loaded with a capacitor, at the input a coil with an inductance equal to $L=C/gm^2$ is seen:

$$Z_{in}(s)\Big|_{Z_L=1/(sC)} = \frac{1}{gm^2}\frac{1}{1/(sC)} = sL$$

How to realize a gyrator? Using equations of the gyrator, e.g. directly, through the use of transconductance amplifiers.

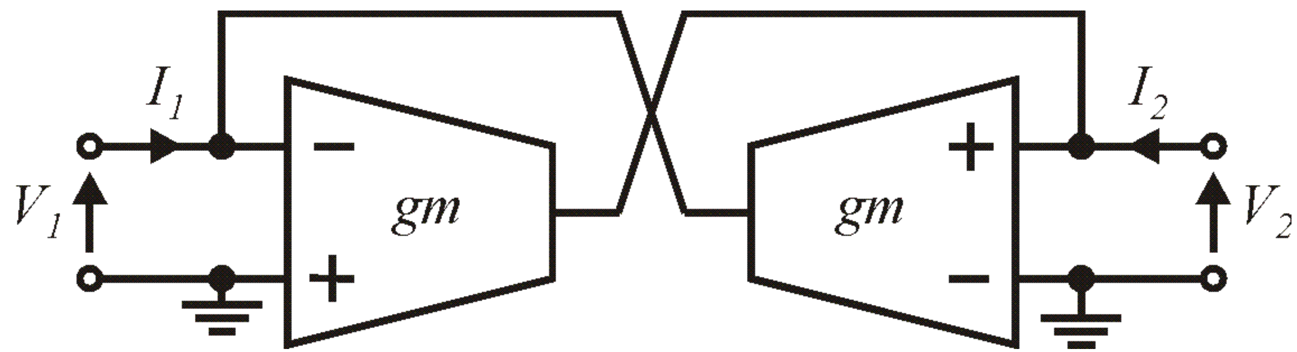


Fig. 59. OTA implementation of a grounded gyrator.

Direct simulation of RLC prototype – gyrator, cont.

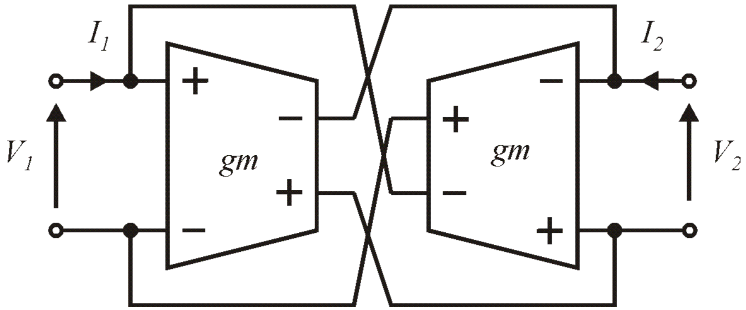
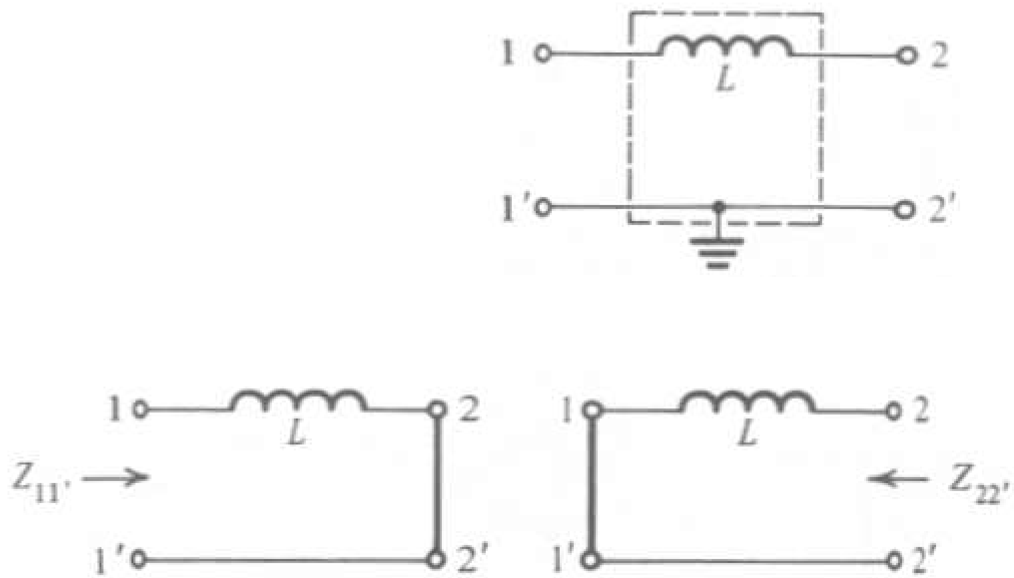


Fig. 60. Fully-differential OTA implementation of a non-grounded gyrator.



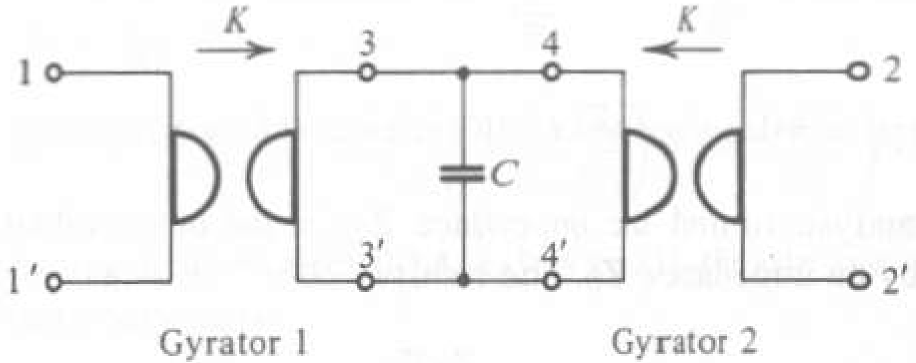


Fig. 61. Realization of a non-grounded coil using 2 gyrators [1].

Direct simulation of RLC prototype – gyrator, cont.

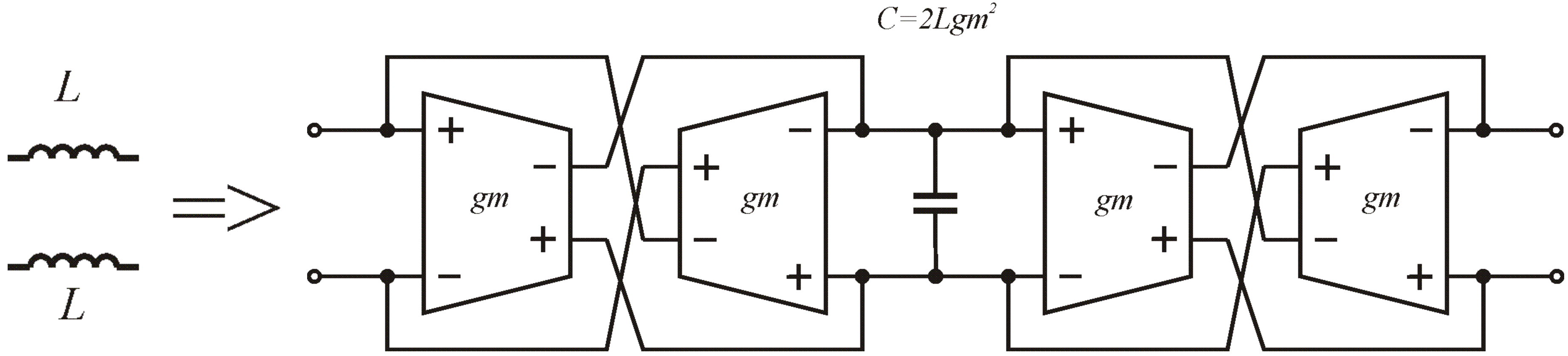


Fig. 62. Implementation of non - grounded symmetrical coils using two fully-balanced OTAs.

Homework no 4

How can the gyrator be realized using operational amplifiers and second generation current conveyors? Please provide schematics and values of gyration conductance / resistance factors.

Direct simulation of RLC prototype – summary

Summary of the technique:

• devices that can be directly implemented are carried out without changes, examples: capacitors, sometimes resistances,

• the remaining devices are realized by active simulation, examples: L => gyrator + C, R => OTA in the resistor connection ...

• the real devices values are calculated by denormalization of the RLC prototype, it is based on known filter characteristic frequency and possible ranges of realizable values of active and passive devices.

Calculation example no 6

Please design a lowpass, V-order, Butterworth approximation, fully differentional OTA-C filter with 3dB passband frequency equal to 20MHz. Direct simulation of RLC ladder prototype should be used. OTAs with 50μS transconductance are available.

Solution: one can find in Fig. 57 the values of the normalized RLC prototype elements. There are two possible realizations containing 3 coils and 2 capacitors or 3 capacitors and 2 coils. We choose the implementation with a smaller number of coils, because these elements are implemented using the simulation method. The prototype of the RLC filter is thus as below:

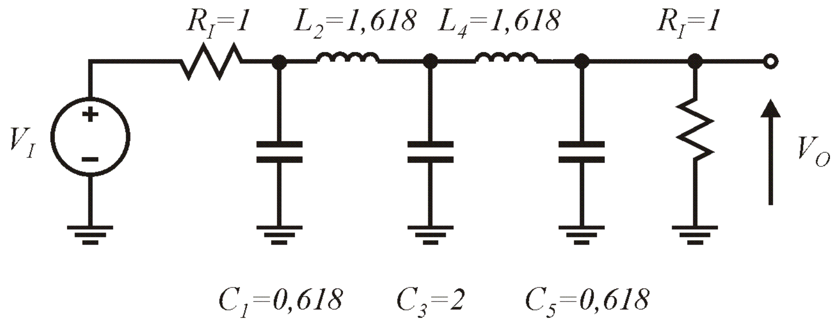


Fig. 63. A normalized RLC ladder prototype of a V-order lowpass filter with a Butterworth approximation [1].

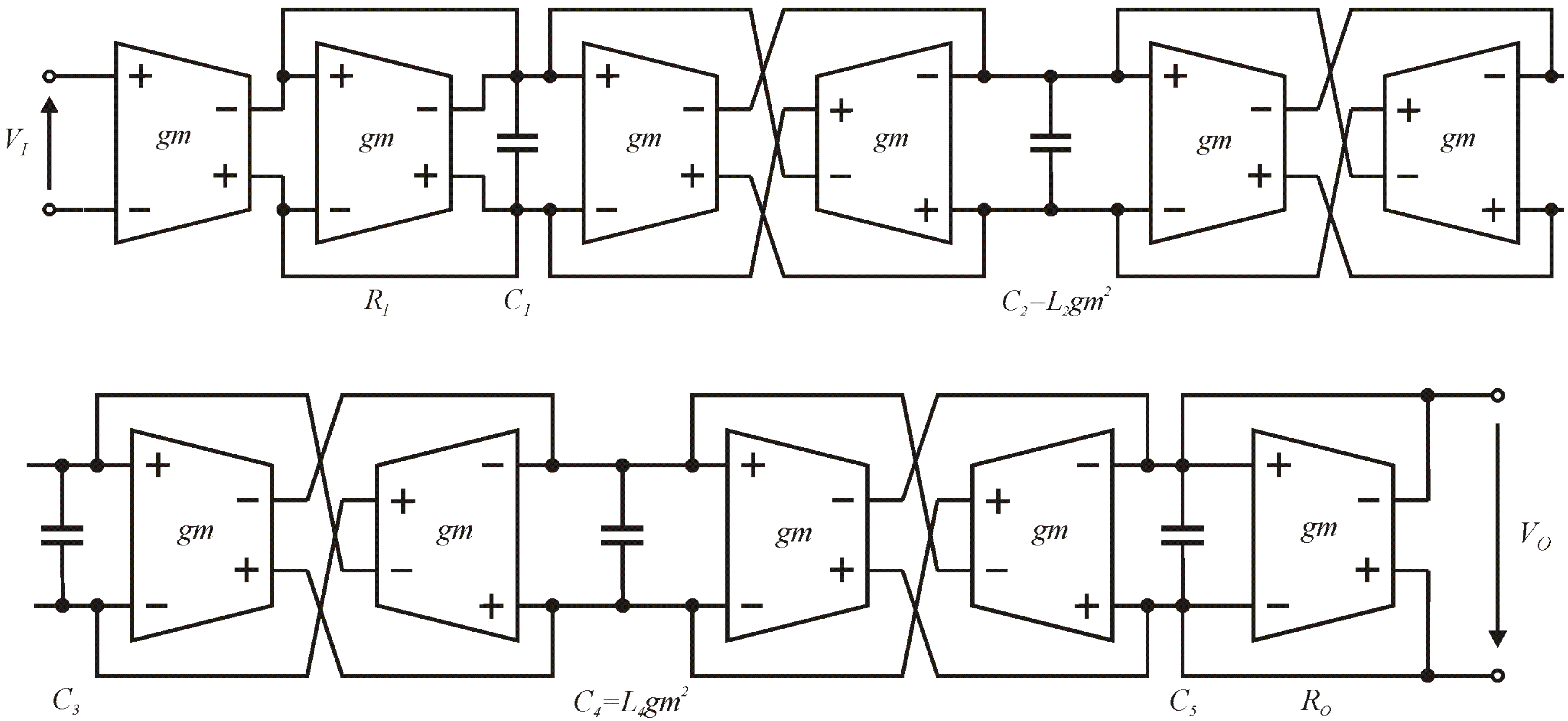


Fig. 64. A fully differentional OTA-C filter implemented by direct RLC ladder prototype simulation method.

The values of individual devices can be calculated on the basis of denormalization:

$$R_S = 1/gm = 1/50\mu S = 20k\Omega \qquad \omega_S = 2\pi 20MHz = 125{,}66Mrad/\sec$$

$$C_1 = \frac{1}{R_S\omega_S}C_{1n} = \frac{50\mu S}{125{,}66Mrad/\sec}\cdot 0{,}618 = 245{,}9\,fF$$

$$C_3 = \frac{1}{R_S\omega_S}C_{3n} = \frac{50\mu S}{125{,}66Mrad/\sec}\cdot 2 = 795{,}8\,fF$$

$$C_5 = \frac{1}{R_S\omega_S}C_{5n} = \frac{50\mu S}{125{,}66Mrad/\sec}\cdot 0{,}618 = 245{,}9\,fF$$

$$C_2 = L_2 gm^2 = \frac{1}{gm\cdot\omega_S}L_{2n}\cdot gm^2 = \frac{gm}{\omega_S}L_{2n} = \frac{50\mu S}{125{,}66Mrad/\sec}\cdot 1{,}618 = 643{,}8\,fF$$

$$C_4 = L_4 gm^2 = \frac{1}{gm\cdot\omega_S}L_{4n}\cdot gm^2 = \frac{gm}{\omega_S}L_{4n} = \frac{50\mu S}{125{,}66Mrad/\sec}\cdot 1{,}618 = 643{,}8\,fF$$

General simulation of signal flow graph of the RLC ladder prototype [1]

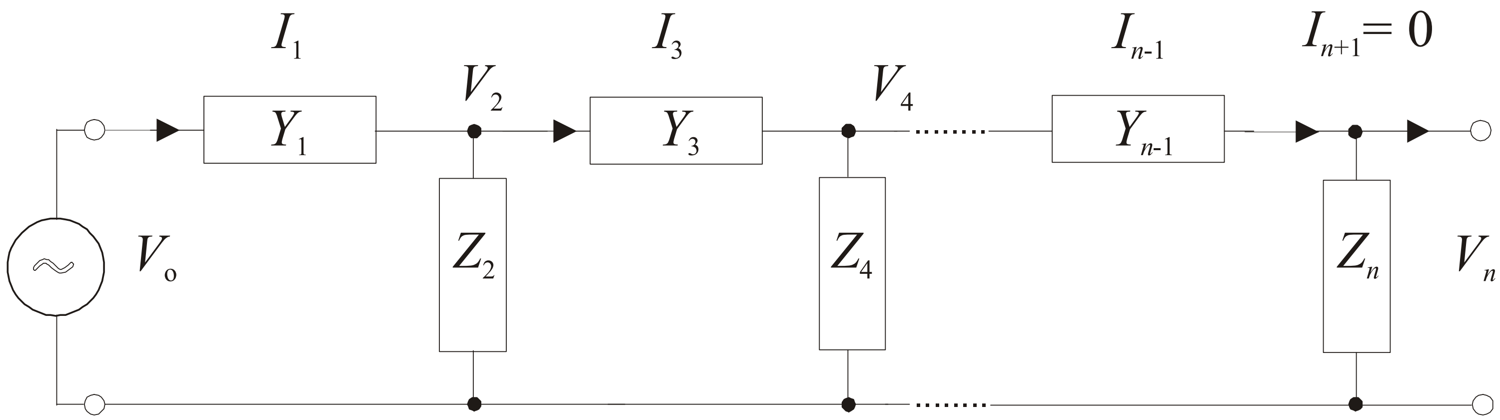


Fig. 65. Immitance ladder representing the RLC filter.

Fig. 65 shows a typical immitance ladder of the RLC filter. The number of *Z*/*Y* devices is *n*. In the case of the odd order of the filter, i.e. when $m=n$-1, the device Y_1 or Z_n must represent only the resistance. The admittance devices have odd indexes, and impedance one have even indexes. For the distinction they will be marked with the letters *i* and *j* respectively. Input and output voltages are denoted by V_0 and V_n, respectively. According to Fig. 65, current flowing through *Y* branches and voltage on *Z* branches can be represented by the following formulas:

$$I_1(s) = \left(V_0(s) - V_2(s)\right)Y_1 \qquad V_2(s) = \left(I_1(s) - I_3(s)\right)Z_2$$

$$I_3(s) = \left(V_2(s) - V_4(s)\right)Y_3 \qquad V_4(s) = \left(I_3(s) - I_5(s)\right)Z_4$$

$$\ldots$$

$$I_{n-1}(s) = \left(V_{n-2}(s) - V_n(s)\right)Y_{n-1} \qquad V_j(s) = \left(I_{j-1}(s) - I_{j+1}(s)\right)Z_j$$

The above equations can be rewritten in the general form:

$$I_i(s) = \left(V_{i-1}(s) - V_{i+1}(s)\right)Y_i \qquad V_j(s) = \left(I_{j-1}(s) - I_{j+1}(s)\right)Z_j$$

Those relationships are valid for odd values of i in the range from 1 to n-1 and even j values in the range from 2 to n, with $I_{n+1}(s)=0$. In each of the above equations, there are currents and voltages related to the respective branches. We multiply those equations and divide them by the resistive scaling factor R_* and get:

$$R_* I_i(s) = \left(V_{i-1}(s) - V_{i+1}(s)\right)Y_i R_* \qquad V_j(s) = \left(I_{j-1}(s)R_* - I_{j+1}(s)R_*\right)Z_j / R_*$$

Marking further as:

$$R_* I_i(s) = V_{Ii}(s) \qquad Y_i R_* = T_{Yi} \qquad Z_j / R_* = T_{Zj}$$

we get:

$$V_{Ii}(s) = \left(V_{i-1}(s) - V_{i+1}(s)\right) T_{Yi} \qquad V_j(s) = \left(V_{Ij-1}(s) - V_{Ij+1}(s)\right) T_{Zj}$$

And then we can accomplish these equations using a voltage signal graph as shown in the figure below.

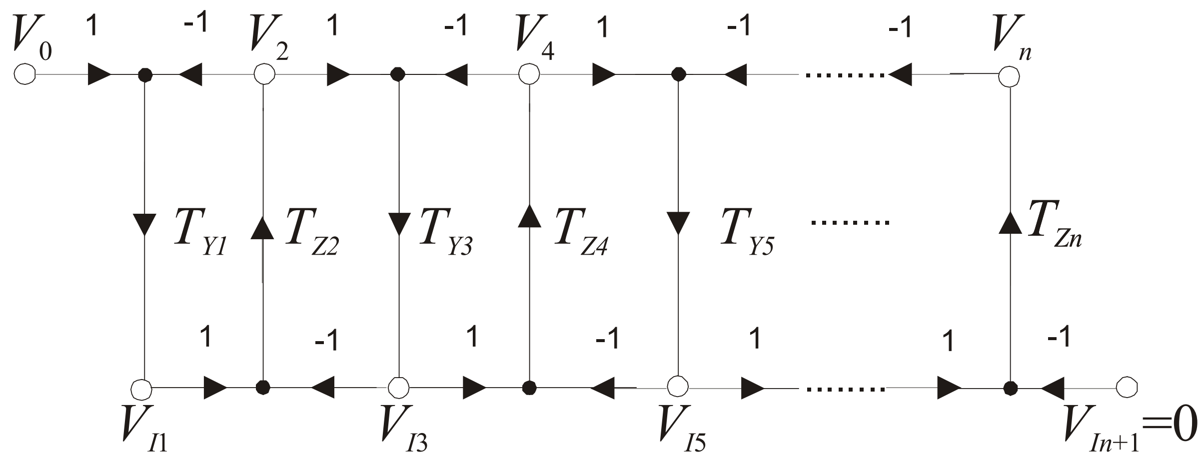


Fig. 66. Signal flow graph realizing the simulation of the filter prototype from Fig. 65.

Simulation of the signal flow graph of the RLC ladder lowpass prototype [1]

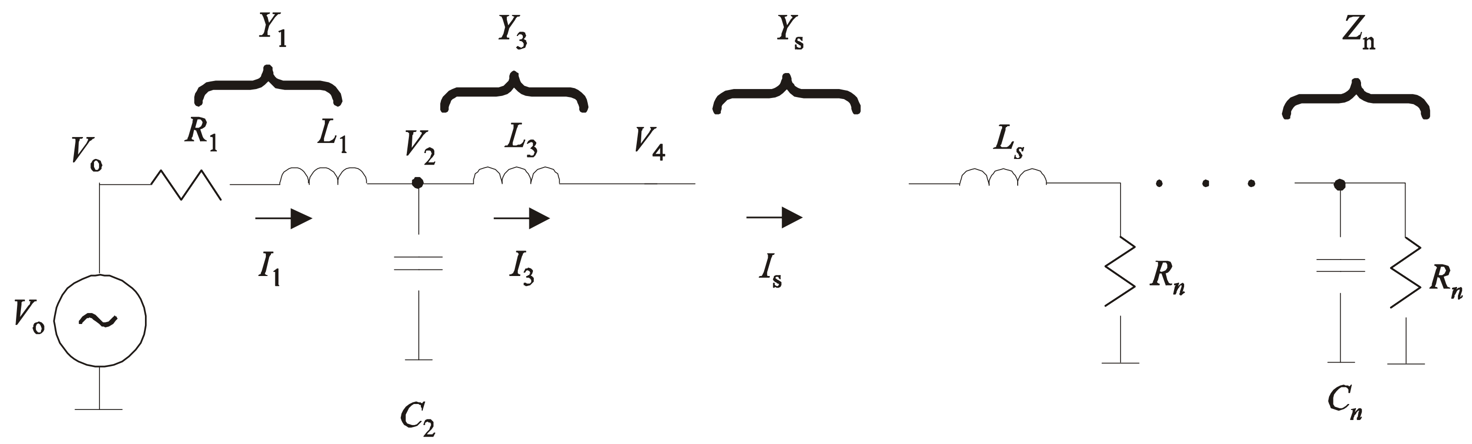


Fig. 67. Lowpass RLC filter.

The lowpass filter can contain serial R-L circuits in horizontal branches and parallel R-C in vertical branches. Therefore, the corresponding values of the lowpass filter immitances can be expressed by the following relationships:

$$Z_j(s) = \frac{1}{sC_j + 1/R_j} \qquad Y_i(s) = \frac{1}{sL_i + R_i}$$

In the absence of resistive elements, the equations above take the form:

$$Y_i(s) = \frac{1}{sL_i} \qquad Z_j(s) = \frac{1}{sC_j}$$

After multiplying / dividing by the scaling resistance R_* we get transfer functions:

$$T_{Yi} = Y_i(s)R_* = \frac{1}{sL_i / R_* + R_i / R_*} \qquad T_{Zj} = Z_j(s) / R_* = \frac{1}{sC_j R_* + R_* / R_j}$$

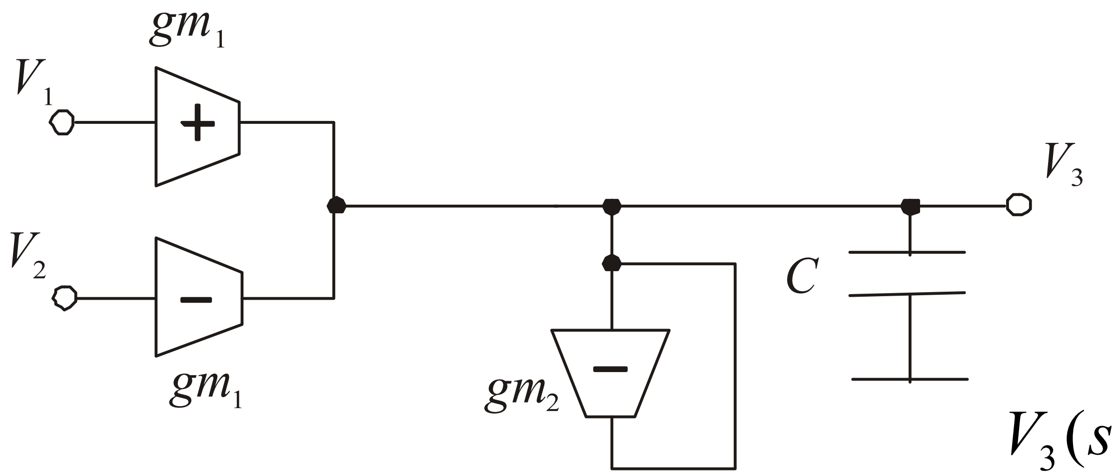
$$T_{Yi} = Y_i(s)R_* = \frac{1}{sL_i / R_*} \qquad T_{Zj} = Z_j(s) / R_* = \frac{1}{sC_j R_*}$$

The above transfer function represent lossy or ideal integrators for branches with or without resistances, respectively.

The lowpass RLC filter can be implemented using integrators and active adders!

OTA-C implementation of lowpass signal flow graph simulation technique

Practical OTA-C filters are usually made using differential amplifiers. In order to simplify the schematic diagrams of presented implementations here grounded OTAs are used.



$$V_3(s) = \left(V_1(s) - V_2(s)\right)\frac{gm_1}{sC + gm_2}$$

Fig. 68. Implementation of the summing and integrating branch using OTA amplifiers and capacitors.

Comparing the previous expressions with the description of the circuit from Fig. 68 and using the denormalization of the RLC prototype, the T_{Yi} transfer functions can be realized in the manner shown in Fig. 69. In this case, the system parameters can be calculated using:

$$\frac{gm_{1i}}{C_{Gi}} = \frac{R_* \omega_S}{L_{Pi} R_S} \qquad gm_{2i} = gm_{1i} \frac{R_S}{R_*} R_{Pi}$$

where: gm_{1i}, gm_{2i}, C_{gi} – values of devices in corresponding i branch , R_* - scaling resistance, R_S – normalizing resistance.

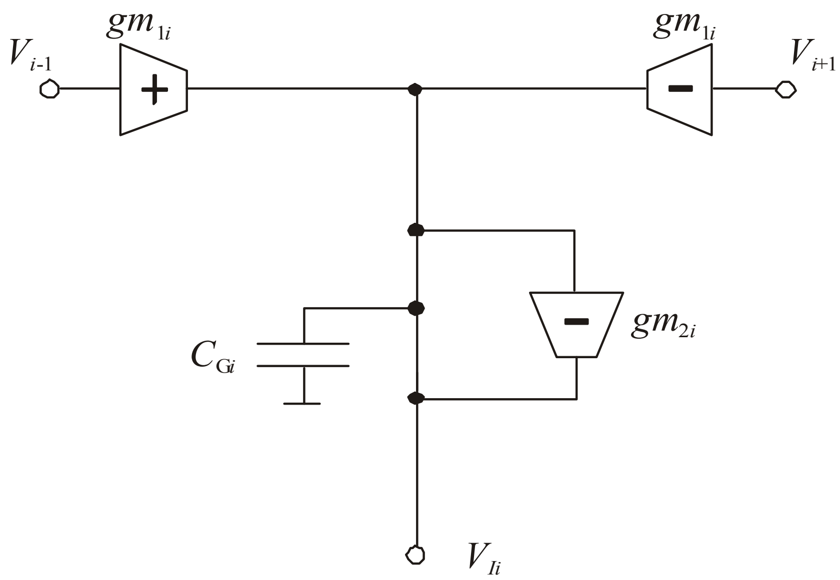


Fig. 69. T_{Yi} branch realisation of transfer function for lowpass filter.

Similarly as above, for T_{Zj} we can get:

$$\frac{gm_{1j}}{C_{Gj}} = \frac{R_S \omega_S}{C_{Pj} R_*} \qquad gm_{2j} = gm_{1j} \frac{R_*}{R_S R_{Pj}}$$

where: gm_{1j}, gm_{2j}, C_{Gj}– values of devices in corresponding j branch, R_* - scaling resistance, R_S – normalizing resistance.

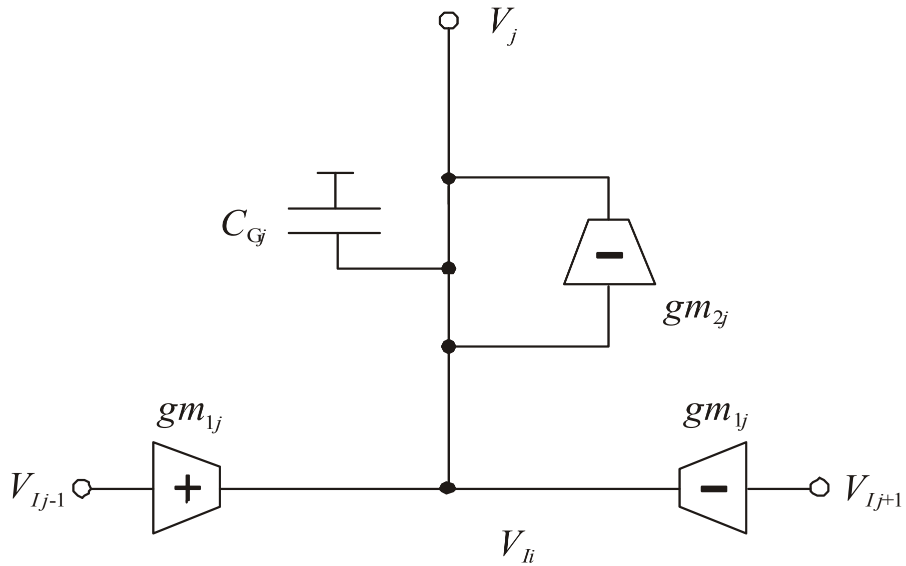


Fig. 70. T_{Zj} branch realisation of transfer function for lowpass filter.

Calculation example no 7

Please design a fully differentall, lowpass, V-order filter, Butterworth approximation with 3dB passband equal to 20MHz. The RLC ladder prototype signal flow graph simulation technique should be used. OTAs with transconductance equal to 50μS are available.

Solution: we find in Fig. 57 the values of the standardized RLC ladder prototype devices. There are two possible realizations containing 3 coils and 2 capacitors or 3 capacitors and 2 coils. We choose any implementation, e.g. like this one below:

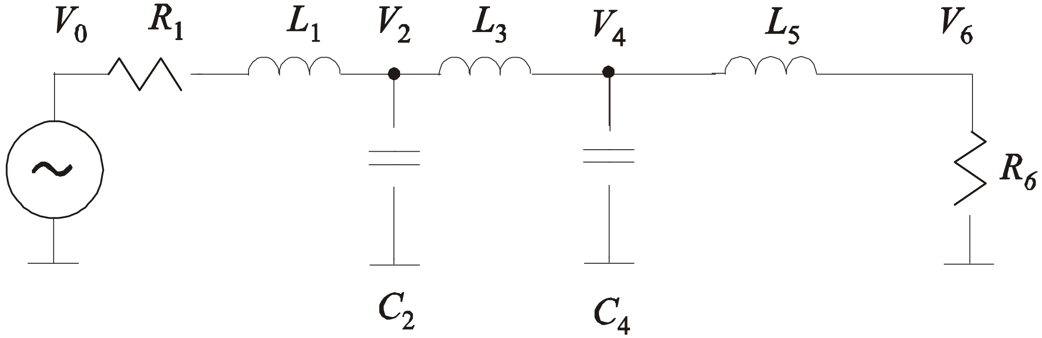


Fig. 71. RLC ladder prototype of the V-order lowpass filter.

On the basis of Fig. 69 and Fig. 70, we set the final schematic of the filter as in Figure 72.

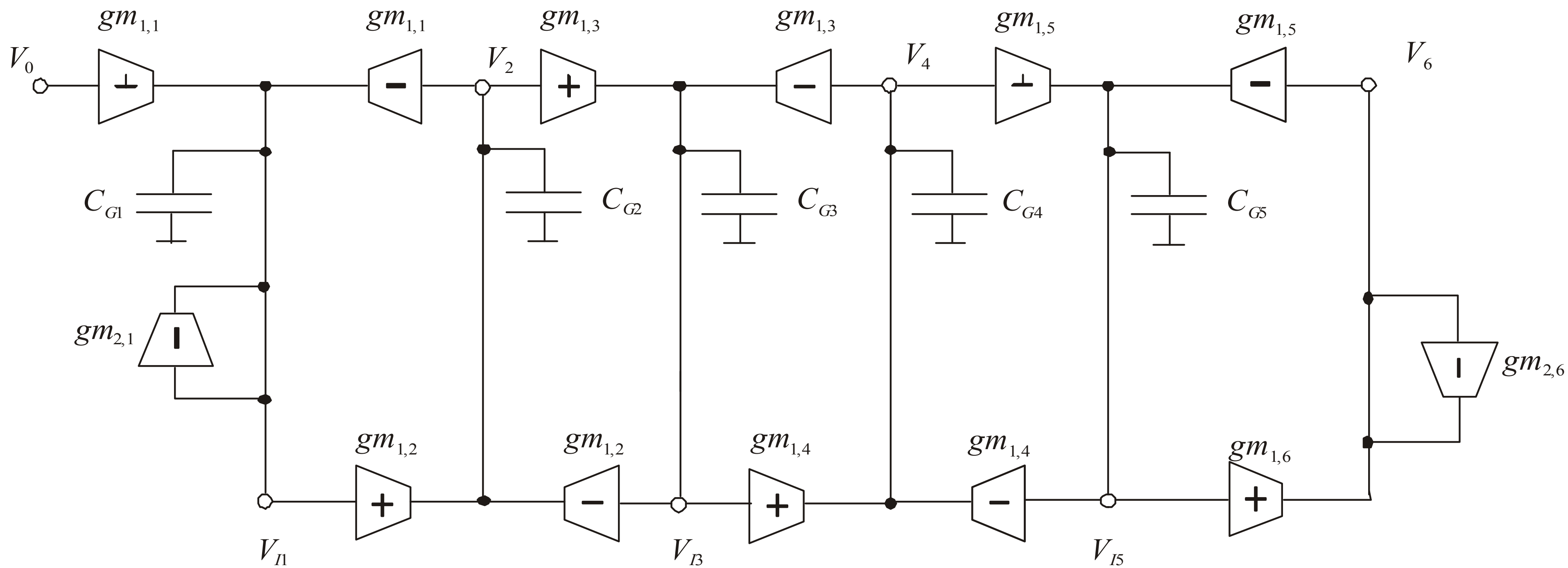


Fig. 72. OTA-C implementation of V-order, lowpass filter using signal flow graphs simulation. In place of amplifiers, both symmetrical and grounded OTAs can be used.

To determine the final values of devices, we assume that the normalizing resistance is equal to the scaling resistance R_S=R_*, using transconductance of value $gm=50\mu$S for all OTAS we get values:

$$R_S = 1/gm = 1/50\mu S = 20k\Omega \qquad \omega_S = 2\pi 20MHz = 125{,}66 Mrad/\sec$$

$$C_{G1} = \frac{gm}{\omega_S} \cdot L_{P1} = \frac{50\mu S}{125{,}66 Mrad/\sec} \cdot 0{,}618 = 245{,}9\, fF$$

$$C_{G2} = \frac{gm}{\omega_S} \cdot C_{P2} = \frac{50\mu S}{125{,}66 Mrad/\sec} \cdot 1{,}618 = 643{,}8\, fF$$

$$C_{G3} = \frac{gm}{\omega_S} \cdot L_{P3} = \frac{50\mu S}{125{,}66 Mrad/\sec} \cdot 2 = 795{,}8\, fF$$

$$C_{G4} = \frac{gm}{\omega_S} \cdot C_{P4} = \frac{50\mu S}{125{,}66 Mrad/\sec} \cdot 1{,}618 = 643{,}8\, fF$$

$$C_{G5} = \frac{gm}{\omega_S} \cdot L_{P5} = \frac{50\mu S}{125{,}66 Mrad/\sec} \cdot 0{,}618 = 245{,}9\, fF$$

Frequency transformations [1]

So far, the lecture presents techniques of approximation and synthesis of lowpass filters. Frequency transformations allow the transfer of known techniques for lowpass filters to filters with different characteristics. There are many different frequency transformations known and here the two basic ones will be presented:

- LP – HP transformation,
- LP – BP transformation.

The symbolism used will be as follows:

- coordinates associated with the LP prototype will use capital letters $S=\Sigma+j\Omega$,
- coordinates for the target filter will use lower case letters $s=\sigma+j\omega$.

Frequency transformations [1]-cont.

The problem that we want to solve can be presented as follows:

- we have transfer function $T_L(S)$ with known and desirable parameters,
- we are looking for a function $\boldsymbol{X}$ transforming this transfer function to a highpass, bandpass or other desired transfer function:

$$\boldsymbol{\Omega = X(\omega)}$$

- we want to transform only the horizontal axis of the $|\boldsymbol{T_L(j\Omega)}|$ module, without any change in the vertical axis,
- it must also be remembered that transmittance $\boldsymbol{T_L(j\Omega)}$ is an even function of frequency, also defined for negative values.

LP – HP transformation [1,5]

Transformation is made by substitution:

$$\Omega = -\frac{1}{\omega}$$

What for the complex variable corresponds to the transformation:

$$S = \frac{1}{s}$$

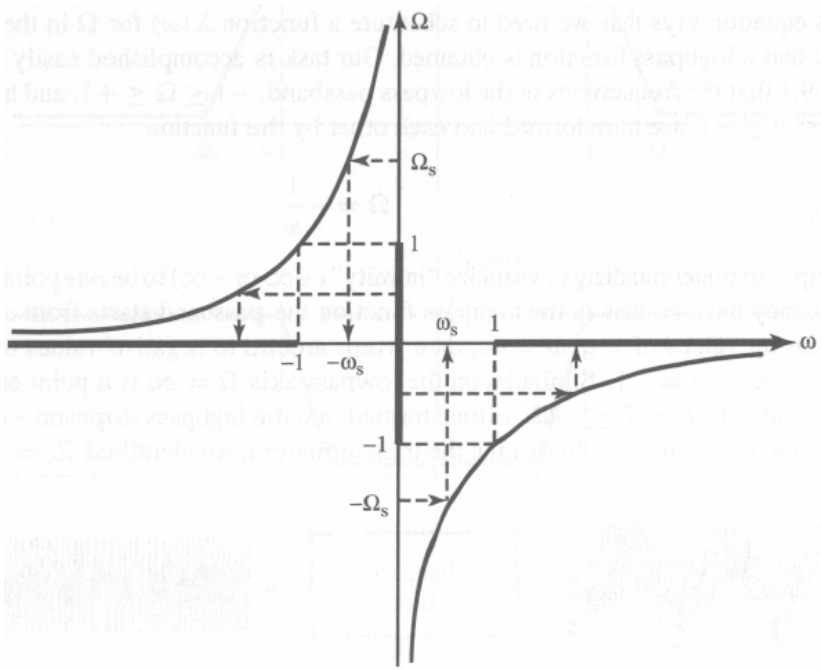


Fig. 73. Frequency change in LP - HP transformation [1].

The procedure of highpass filter design using LP-HP transformation:

- frequency normalization of the highpass filter,
- transfer of the characteristic frequencies to the lowpass form using the rule of inverting frequency values (omitting the sign), this is shown in Fig. 74,
- selection of the transfer function of an appropriate filter that meets requirements of lowpass filter (approximation),
- determination of highpass filter transfer function by substitution S=1/s,
- realization of the determined transfer function (synthesis).

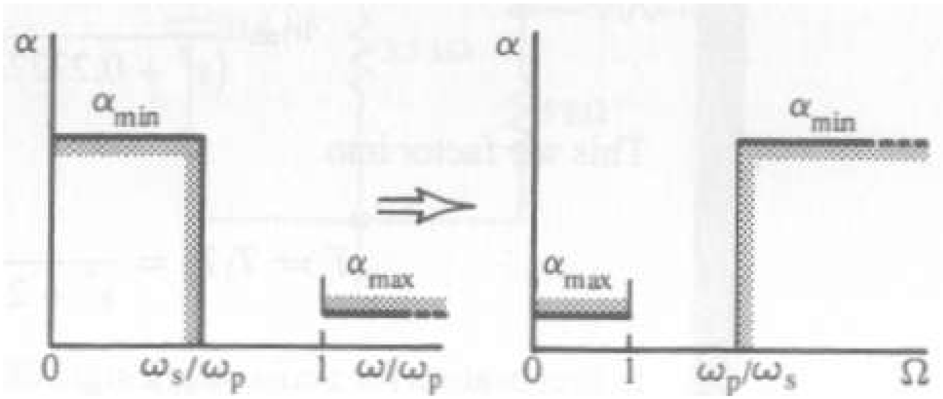


Fig. 74. Transformation of highpass filter requirements into the equivalent lowpass one [1].

Remarks regarding realization of the new, highpass transfer function

In the case of cascade realization using biquad sections, the LP transmittance changes as follows:

$$T_{LP}(S) = H\frac{\Omega_O^2}{S^2 + \frac{\Omega_O}{Q}S + \Omega_O^2} \quad \text{for } S = 1/s \text{ changes into:}$$

$$T_{HP}(s) = H\frac{\Omega_O^2}{\frac{1}{s^2} + \frac{\Omega_O}{Q}\frac{1}{s} + \Omega_O^2} = H\frac{s^2}{s^2 + \frac{\omega_O}{Q}s + \omega_O^2}$$

where: $\Omega_O = \dfrac{1}{\omega_O}$

Hence the conclusion that in the case of a cascaded filter realization, frequency transformation converts LP biquads to HP ones.

Remarks regarding realization of the new, highpass transfer function

In the case of RLC ladder prototype simulation, the devices are replaced, which can be determined by comparing the immitances:

$$Y_{C,LP} = SC \text{ for } S = 1/s \text{ changes to admitance } Y_{C,HP} = \frac{1}{s}C$$

$$Z_{L,LP} = SL \text{ for } S = 1/s \text{ changes to impedance } Z_{C,HP} = \frac{1}{s}L$$

Hence the conclusion that capacitor C in the LP filter turns into a 1/C coil in the HP filter. Similarly, the L coil in the LP filter turns into a 1/L capacitor in the HP filter.

The remaining devices are unchanged.

LP – HP transformation does not change order of the filter.

Calculation example no 8, highpass filter design

Please design a highpass filter which should attenuate less than 3dB signals with frequencies above 50kHz and more than 40dB signals with frequencies below 12.5kHz.

Solution: we carry out the procedure in accordance with the principles of using the LP => HP transformation:

• normalization: 50kHz -> 1rad/sec, 12,5kHz ->0,25rad/sec,

• transformation of requirements to LP: attenuation for 1rad/sec remains on 3dB, while attenuation of 40dB for 0.25rad/sec turns into attenuation equal to 40dB for 1/0.25 = 4rad/sec,

• we use Butterworth approximation, from the graph on page 104 we find a filter order of at least 4, When implemented in a cascade form, we get two HP biquad sections with Q=0.54 and Q=1.31 (table on page 103) and a natural frequency of 50kHz.

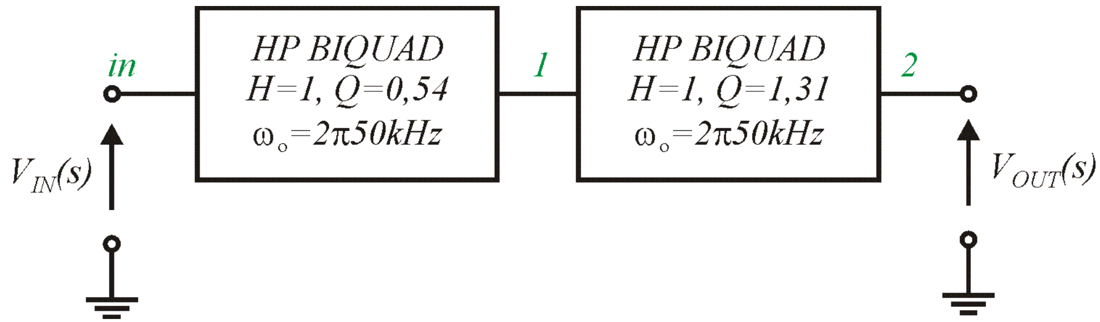


Fig. 75. Cascade realization of highpass filter.

When implementing based on the RLC ladder, we start with the RLC prototype as shown below (from page 114, fig. 57).

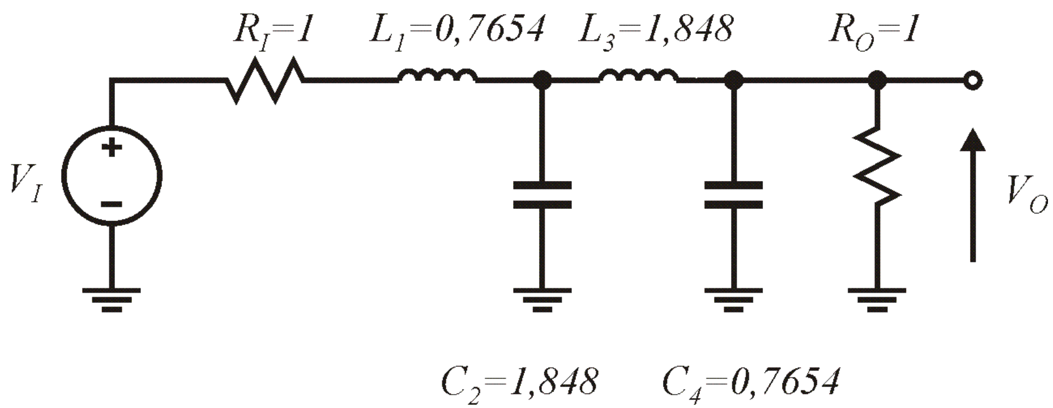


Fig. 76. LP RLC IV-order Butterworth prototype [1].

Then we change the LP prototype to the HP one, what gives the circuit as below.

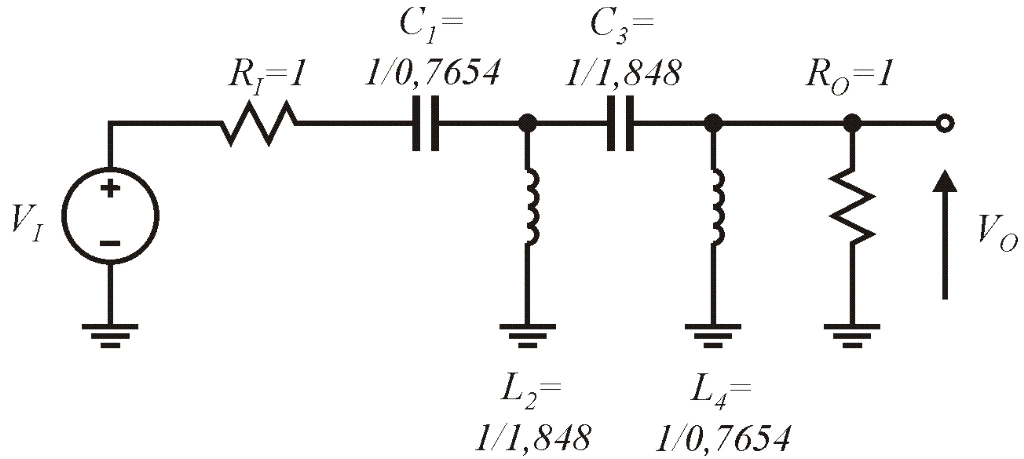


Fig. 77. HP RLC 4th order Butterworth prototype.

Finally, the HP RLC prototype is denormalized and implemented in an active form by any method, e.g. direct prototype simulation or signal flow graph simulation.

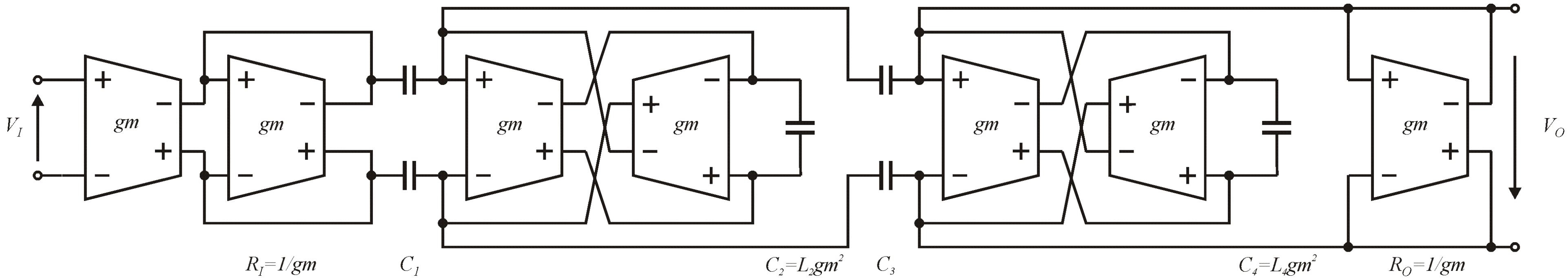


Fig. 78. OTA-C fully differential implementation of 4-th order highpass Butterworth's filter.

Suppose we have OTA amplifiers with *gm*=81,04μS, then the values of individual capacitors can be determined from the equations:

$$\omega_S = 2\pi 50kHz \qquad R_S = 1/gm = 1/81{,}04\mu S$$

$$C_1 = 2\cdot\frac{1}{0{,}7654}\cdot\frac{gm}{\omega_S} = 674{,}85\,pF \qquad C_3 = 2\cdot\frac{1}{1{,}848}\cdot\frac{gm}{\omega_S} = 279{,}18\,pF$$

$$C_2 = L_2 gm^2 = \frac{1}{1{,}848}\cdot\frac{1}{\omega_S gm}gm^2 = \frac{1}{1{,}848}\cdot\frac{gm}{\omega_S} = 139{,}58\,pF$$

$$C_4 = \frac{1}{0{,}7654}\cdot\frac{gm}{\omega_S} = 337{,}02\,pF$$

PSPICE netlist to the calculation example no 8

```
Example No 8
* Parameter settings
.param fo=50k
.param wo={2*3.1415*fo}

* Input source
Vin in 0 dc 0 ac 1
* Input for balanced filter
Ep in_p 0 in 0 0.5
Em in_m 0 in 0 -0.5

* Declaration of ideal HP biquad subcircuit
.subckt biquad_hp_id out in params: wo=1 Q=1
E1 out 0 laplace {V(in)}={s*s/(s*s+wo/Q*s+wo*wo)}
.ends

* Cascaded ideal filter version, output node (2)
X1bq 2 1 biquad_hp_id params: wo={wo} Q=0.54
X2bq 1 in biquad_hp_id params: wo={wo} Q=1.31
RL 2 0 1

* RLC HP denormalized prototype, output node (7)
.param rs={1/gm}
.param gm=81.04u
Ri in 5 {1*rs}
C1 5 6 {1/0.7654/wo/rs}
L2 6 0 {1/1.848/wo*rs}
C3 6 7 {1/1.848/wo/rs}
L4 7 0 {1/0.7654/wo*rs}
Ro 7 0 {1*rs}

* Power supply for OTAs
vdd vdd 0 2.5V
vss vss 0 -2.5V

*OTA subcircuit declaration
*    Name out+  out-  in+  in- Vdd Vss
.sub OTA out_p out_m in_p in_m Vdd Vss
M1 1 in_p 3 vdd pfet w=12u l=4u
M2 2 in_m 3 vdd pfet w=12u l=4u
M3 1 1 vss vss nfet w=2u l=2u
M4 2 2 vss vss nfet w=2u l=2u
M5 out_m 2 vss vss nfet w=2u l=2u m=4
M6 out_p 1 vss vss nfet w=2u l=2u m=4
M7 3 7 4  vdd pfet w=4u l=2u m=2
M8 out_m 7 5 vdd pfet w=4u l=2u m=4
M9 out_p 7 5 vdd pfet w=4u l=2u m=4
M10 7 7 6 vdd pfet w=4u l=2u m=1
M11 4 0 vdd vdd pfet w=2u l=2u m=2
M12 5 out_m vdd vdd pfet w=2u l=2u m=4
M13 5 out_p vdd vdd pfet w=2u l=2u m=4
M14 6 0 vdd vdd pfet w=2u l=2u m=1
Ibias 7 vss 10u
.ends

* OTA-C fully balanced HP filter, output nodes (16,15)
C1gma  11  13  674.85pF
C1gmb  12  14  674.85pF
C2gm   13b 14b 139.58pF
C3gma  13  15  279.18pF
C3gmb  14  16  279.18pF
C4gm   15b 16b 337.02pF
X1 12   11   in_p  in_m Vdd Vss OTA
X2 12   11   11    12   Vdd Vss OTA
X3 16   15   15    16   Vdd Vss OTA
X5 14b  13b  13    14   Vdd Vss OTA
X6 14   13   14b   13b  Vdd Vss OTA
X7 16b  15b  15    16   Vdd Vss OTA
X8 16   15   16b   15b  Vdd Vss OTA

* analysis
.ac dec 100 1k 1000meg
.lib ami_c5.lib
.probe
.end
```

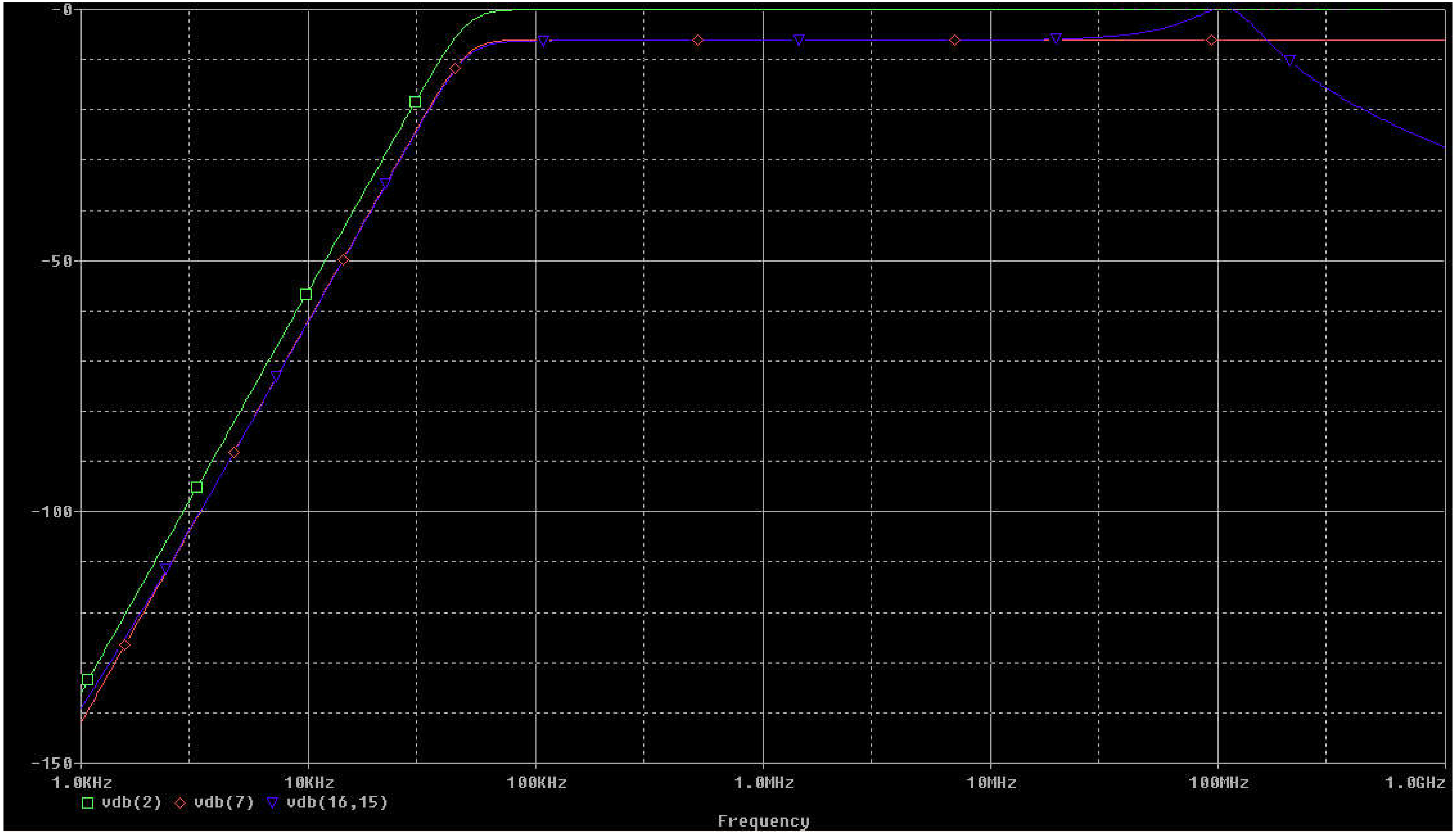


Fig. 79. Results of PSPICE simulations of HP IV-order filter from example no 8, (green) cascade implementation using ideal biquad sections, (red) denormalized HP RLC prototype, (blue) symmetric OTA-C implementation using real CMOS OTA amplifiers (the same as in lab ex. no 1).

LP – BP transformation [1,5]

Rules are the same as for LP=>HP, only the transforming function changes.

$$\Omega = \frac{1}{BW}\frac{\omega^2 - \omega_O^2}{\omega} \quad => \quad \omega_O^2 = \omega_2\omega_1 \qquad BW = \omega_2 - \omega_1$$

For the complex variable S this corresponds to the transformation:

$$S = \frac{1}{BW}\frac{s^2 - \omega_O^2}{s} = \frac{\omega_O}{BW}\frac{s^2 - \omega_O^2}{\omega_O s} = Q\left(\frac{s}{\omega_O} + \frac{\omega_O}{s}\right)$$

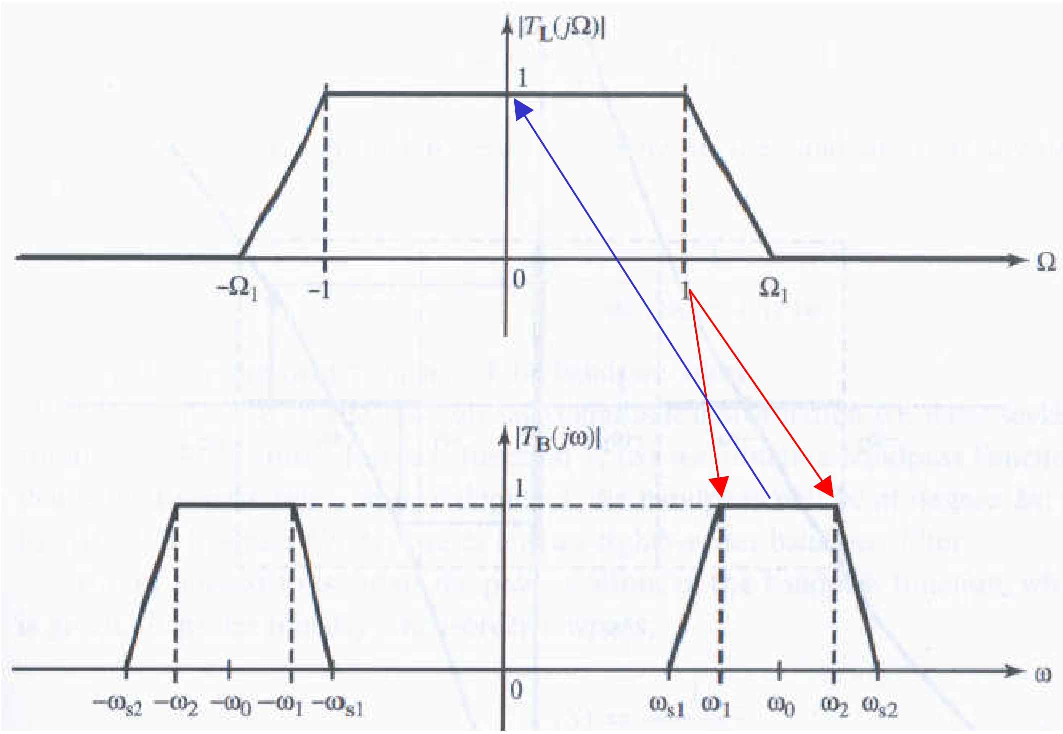


Fig. 80. Replacement of frequency characteristics in the LP=>BP transformation [1].

$$\Omega = \frac{1}{BW}\frac{\omega^2 - \omega_O^2}{\omega}$$

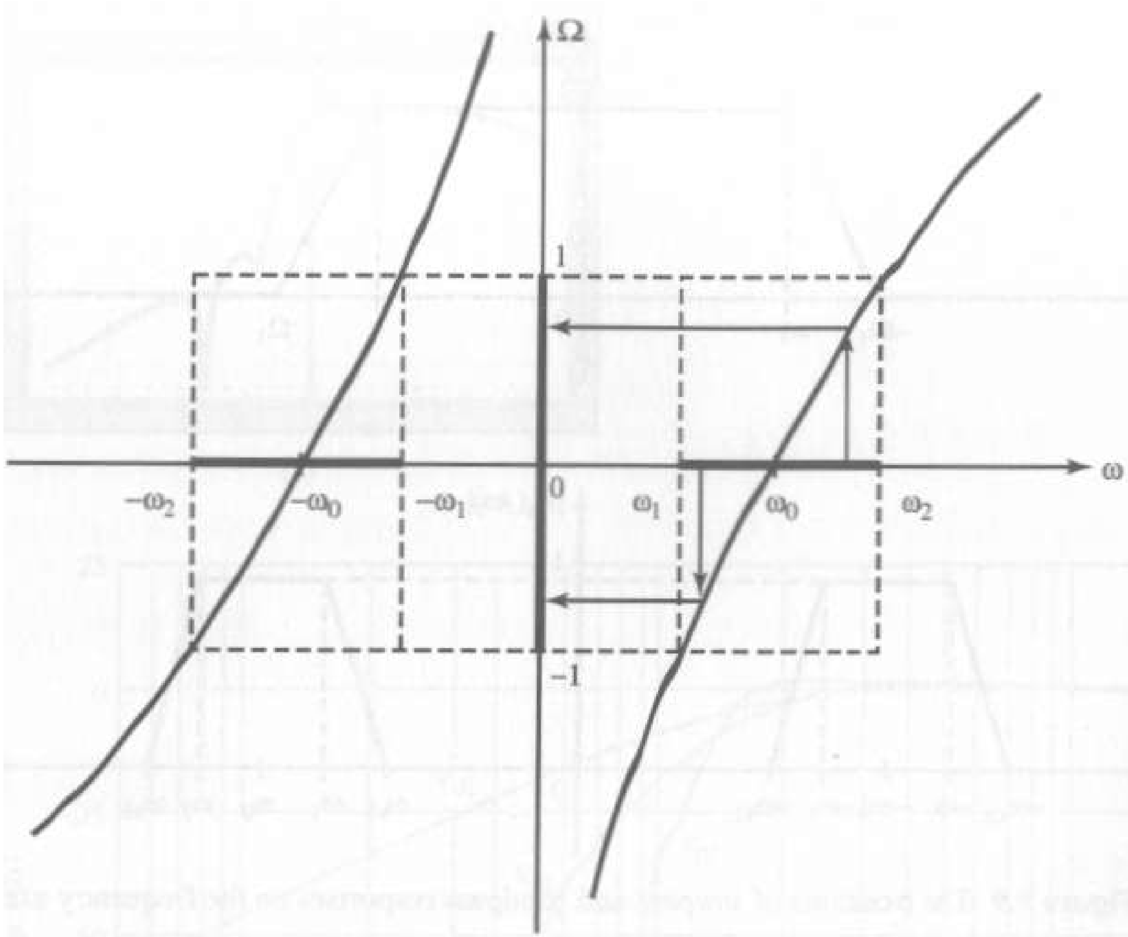


Fig. 81. Transformation of the frequency axis in the LP => BP transformation [1].

The substitution *S=f(s)* for the LP=>BP transformation changes the order of the resultant filter to a twice higher value.

The conversion of the LP filter devices into BP equivalents can be calculated using the comparison of immitances:

$$Y_{C,LP} = SC \text{ for } S = \frac{\omega_O}{BW}\left(\frac{s}{\omega_O} + \frac{\omega_O}{s}\right) \text{corresponds to admitance } Y_{C,BP} = C\frac{s}{BW} + C\frac{\omega_O^2}{BWs}$$

$$Z_{L,LP} = SL \text{ for } S = \frac{\omega_O}{BW}\left(\frac{s}{\omega_O} + \frac{\omega_O}{s}\right) \text{corresponds to impedance } Z_{C,BP} = L\frac{s}{BW} + L\frac{\omega_O^2}{BWs}$$

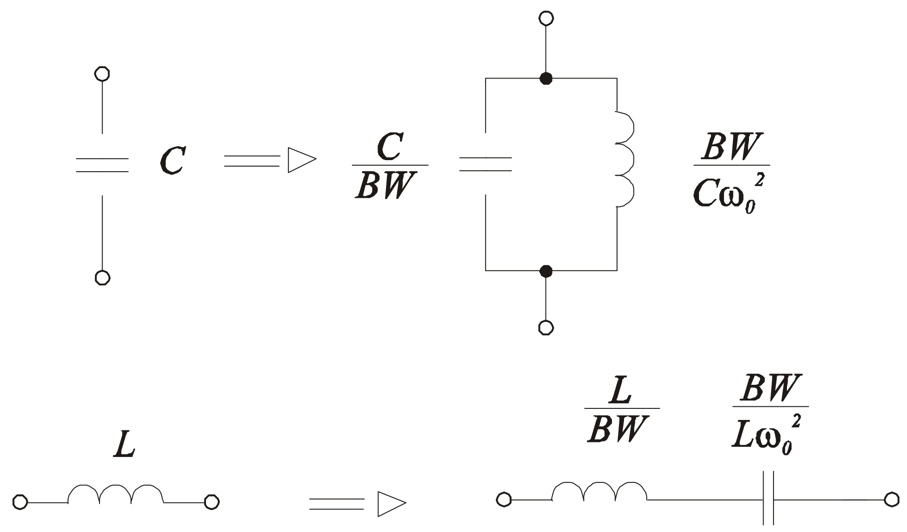


Fig. 82. Replacement of the devices of LP prototype after LP=>BP transformation.

Calculation example no 9, design of bandpass filter

Please design a bandpass filter with an attenuation not greater than 3dB in the 900kHz - 1100kHz band. For frequencies below 800kHz and above 1200kHz filter attenuation should be at least 15dB.

Solution: we perform the procedure as for the HP filter:

Normalization of filter requirements to $\omega_o = 1$:

$$f_O = \sqrt{f_2 f_1} = \sqrt{1100kHz 900kHz} = 995kHz$$

$$\omega_S = 2\pi f_O = 62518 krad/\sec$$

$$f_{1BP} = 900kHz \quad => \quad \omega_{1BP} = 2\pi f_1 / \omega_S = 900kHz / 995kHz = 0{,}9045$$

$$f_{2BP} = 1100kHz \quad => \quad \omega_{2BP} = 2\pi f_2 / \omega_S = 1100kHz / 995kHz = 1{,}1055$$

$$BW = \omega_{2BP} - \omega_{1BP} = 1{,}1055 - 0{,}9045 = 0{,}201$$

$$f_{3BP} = 800kHz \quad => \quad \omega_{3BP} = 2\pi f_3 / \omega_S = 800kHz / 995kHz = 0{,}804$$

$$f_{4BP} = 1200kHz \quad => \quad \omega_{4BP} = 2\pi f_4 / \omega_S = 1200kHz / 995kHz = 1{,}206$$

Transfer of BP normalized frequencies LP frequencies:

$$\Omega = \frac{1}{BW}\frac{\omega^2 - \omega_O^2}{\omega}$$

$$\Omega_1 = \frac{1}{0{,}201}\frac{0{,}9045^2 - 1^2}{0{,}9045} = -1 \qquad \Omega_2 = \frac{1}{0{,}201}\frac{1{,}1055^2 - 1^2}{1{,}1055} = 1$$

$$\Omega_3 = \frac{1}{0{,}201}\frac{0{,}804^2 - 1^2}{0{,}804} = -2{,}188 \qquad \Omega_4 = \frac{1}{0{,}201}\frac{1{,}206^2 - 1^2}{1{,}206} = 1{,}8747$$

In the frequenies listed above, the minus sign should be omitted. The required attenuation for Ω_4 is more stringent than for Ω_3. We choose the approximation of Butterworth and using the drawing on page 104 we can see that in the pulsation range equal to 1.8747, this figure is not precise enough to estimate the required order of the LP filter.

So we use followin equation: $$\left|T_n(j\omega)\right|^2 = \frac{1}{1+\omega^{2n}}$$

$$n = \frac{1}{2}\log_{\omega}\left[\frac{1}{\left|T_n(j\omega)\right|^2} - 1\right] = \frac{1}{2}\log_{1,8747}\left[\frac{1}{\left(10^{-15/20}\right)^2} - 1\right] = \frac{1}{2}\log_{1,8747}(30,623) =$$

$$\frac{1}{2}\frac{\ln 30,623}{\ln 1,8747} = \frac{1}{2}\frac{3,422}{0,6284} = 2,723$$

We choose the integer value n=3. Now it is necessary to implement the Butterworh LP 3 filter properly denormalized and transformed to the BP version. We choose the method of direct simulation of the RLC prototype.

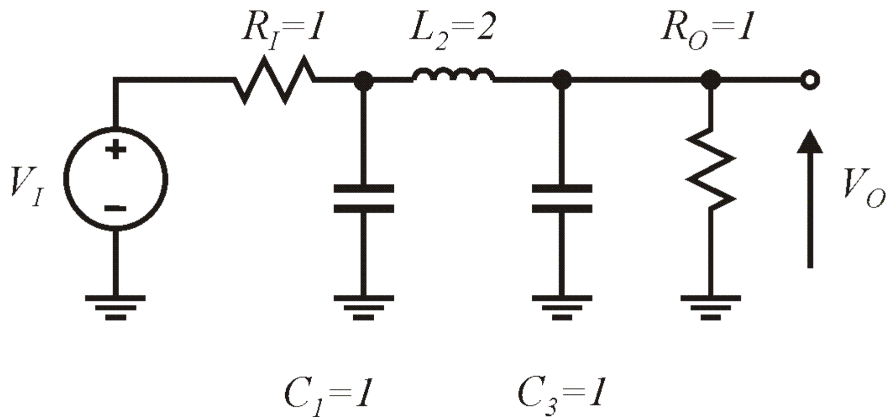


Fig. 83. Butterworth's LP, RLC, III-order normalized prototype [1].

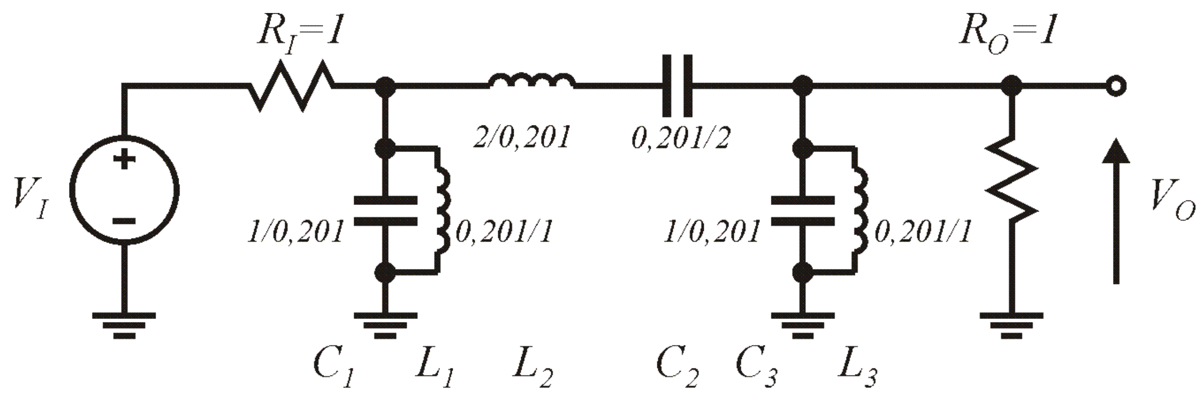


Fig. 84. Butterworth, BP, normalized, VI-order, RLC prototype corresponding to the LP prototope from Fig. 83 for *BW*=0.201.

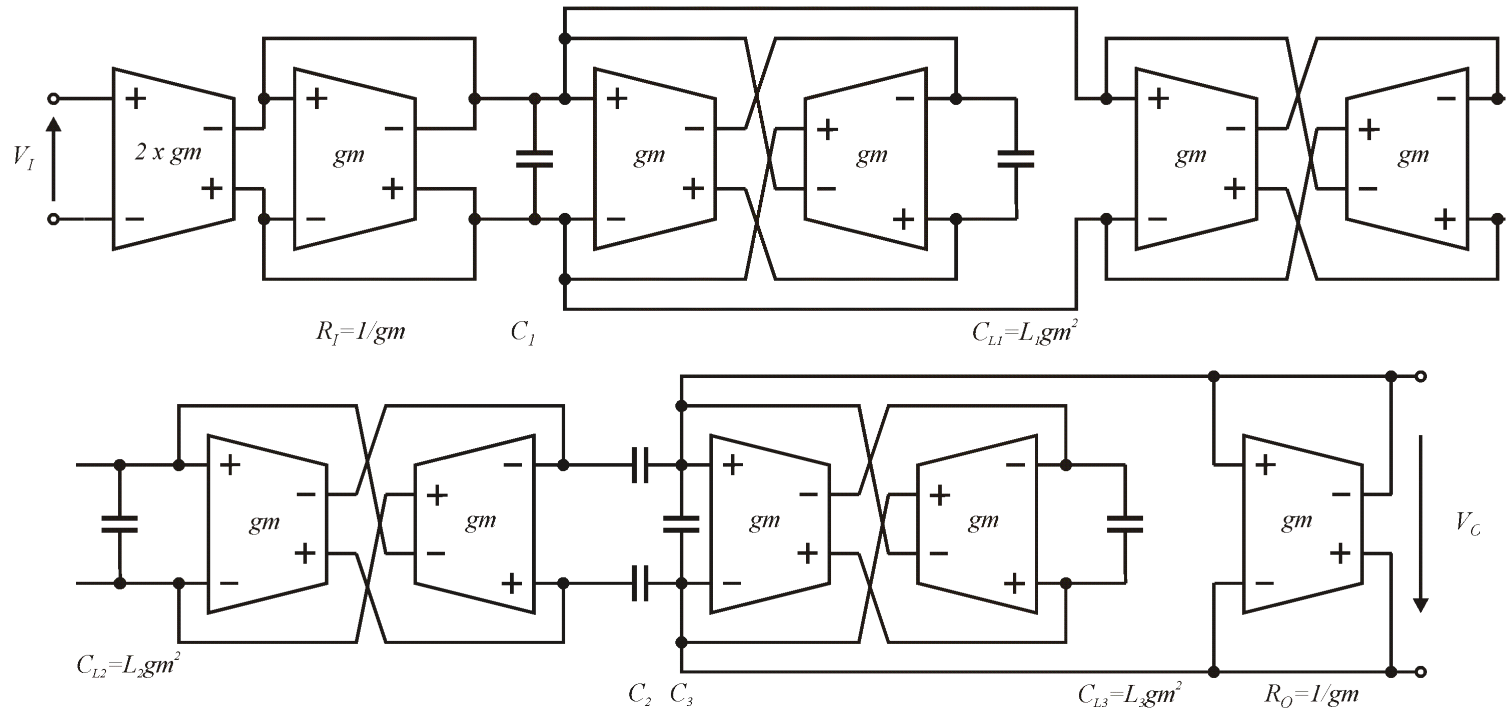


Fig. 85. Fully differentail OTA-C realization of the filter prototype from Fig.84.

Values of capacitors are determined using denormalization procedures, we assume that *gm*=81.04μS:

$$C_1 = \frac{1}{0{,}201} \cdot \frac{gm}{\omega_S} = 64{,}49\,pF \qquad \omega_S = 2\pi f_O = 6251{,}8\,krad/\sec$$

$$C_{L1} = L_1 gm^2 = \frac{0{,}201}{1} \cdot \frac{1}{\omega_S gm} gm^2 = 0{,}201 \cdot \frac{gm}{\omega_S} = 2{,}6055\,pF$$

$$C_2 = 2 \cdot \frac{0{,}201}{2} \cdot \frac{gm}{\omega_S} = 2{,}6055\,pF$$

$$C_{L2} = L_2 gm^2 = \frac{2}{0{,}201} \cdot \frac{1}{\omega_S gm} gm^2 = \frac{2}{0{,}201} \cdot \frac{gm}{\omega_S} = 128{,}98\,pF$$

$$C_3 = \frac{1}{0{,}201} \cdot \frac{gm}{\omega_S} = 64{,}49\,pF$$

$$C_{L3} = L_3 gm^2 = \frac{0{,}201}{1} \cdot \frac{1}{\omega_S gm} gm^2 = 0{,}201 \cdot \frac{gm}{\omega_S} = 2{,}6055\,pF$$

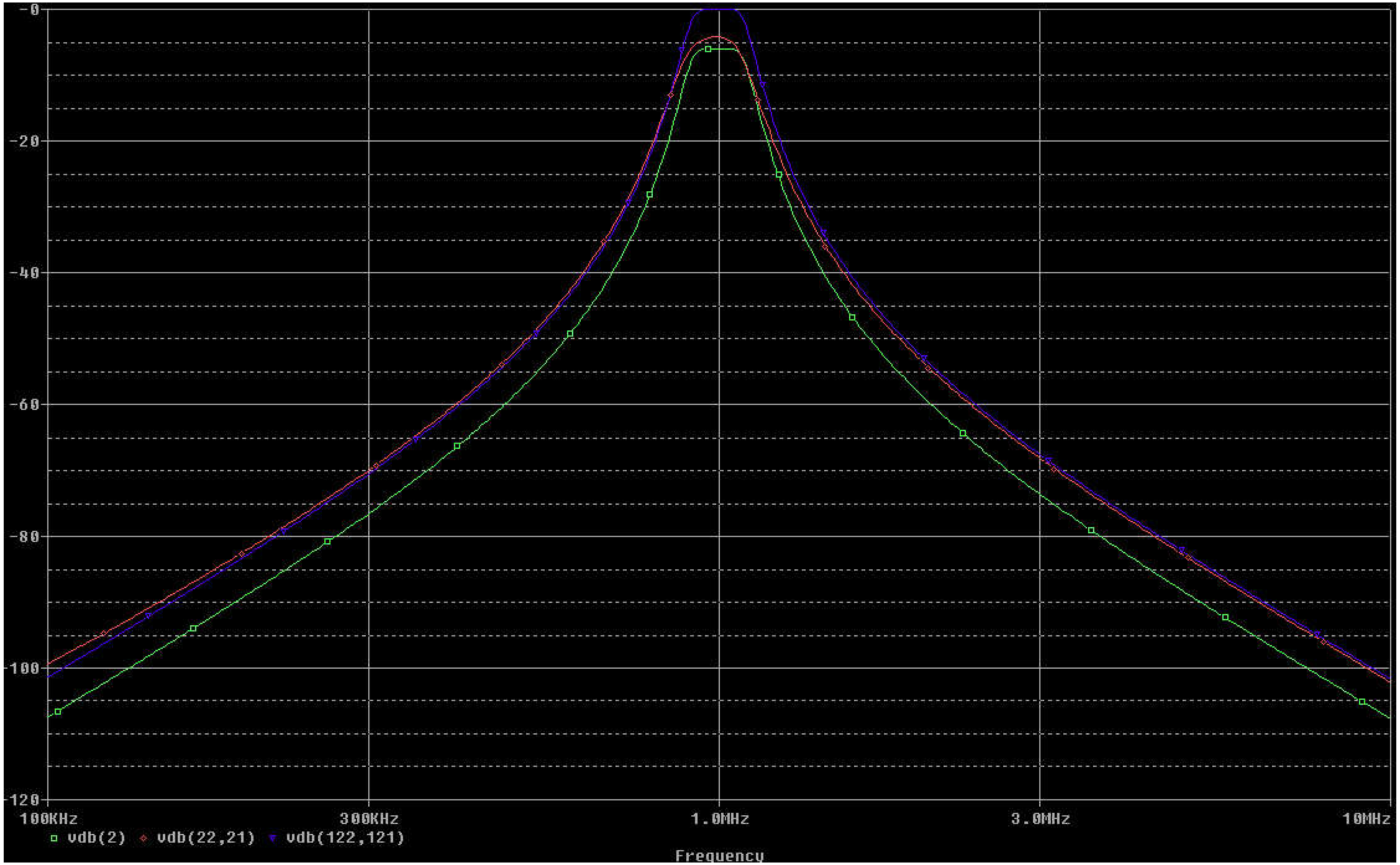
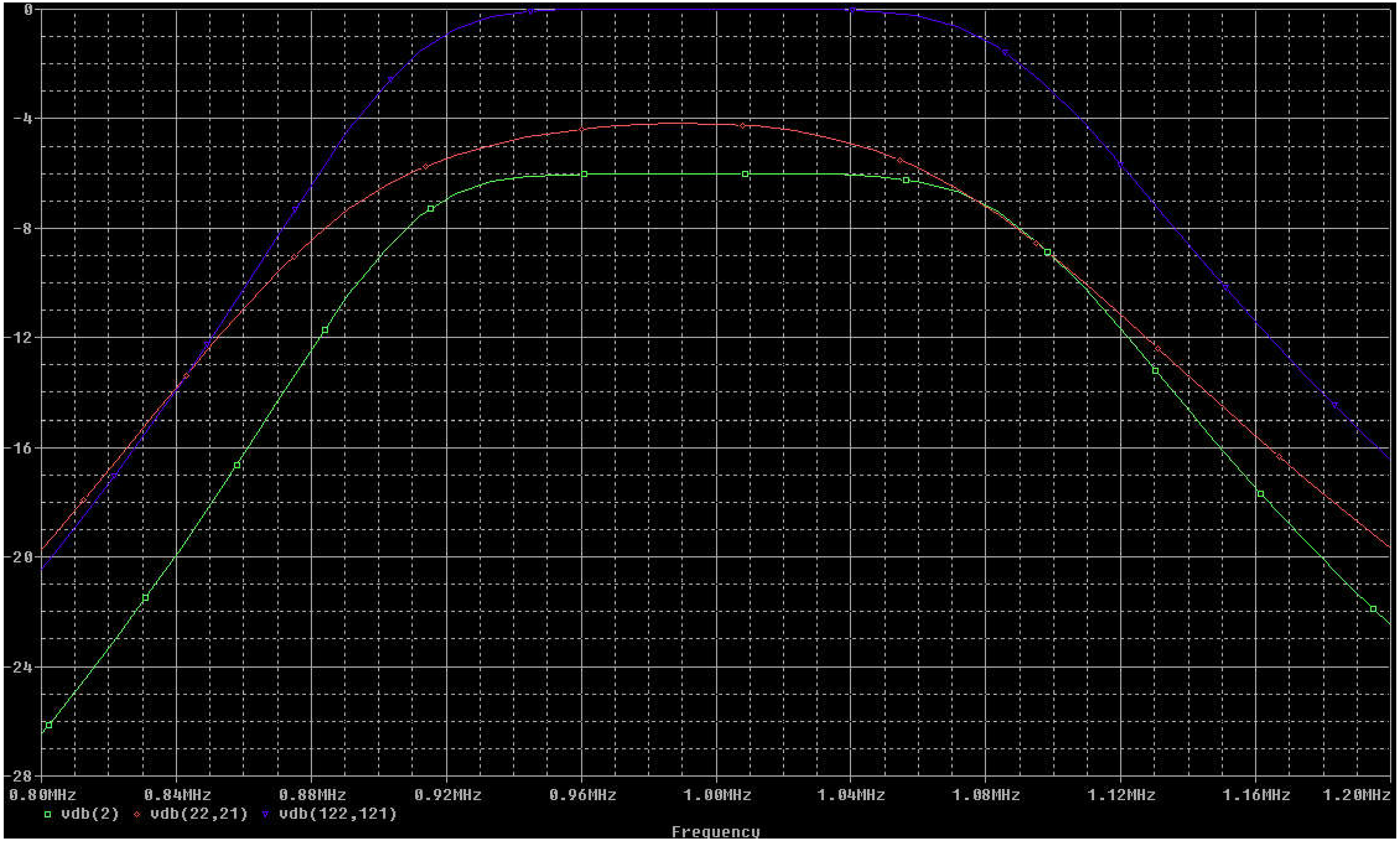


Fig. 86. Results of PSPICE simulation, (green) RLC prototype, (blue) filter as in Fig. 85 with ideal OTA amplifiers and (red) OTA amplifiers as in lab ex. no 1.



Rys. 87. Results of PSPICE simulation zoomed to passband region, (green) RLC prototype, (blue) filter as in Fig. 85 with ideal OTA amplifiers and (red) OTA amplifiers as in lab ex. no 1.